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Optimisation of hybrid off-grid energy systems by linear programming

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Abstract

Background: In this study, a general model of a hybrid off-grid energy system is developed, which can be adjusted to reflect real conditions in order to achieve economical and ecological optimisation of off-grid energy systems.

Methods: Using linear programming methods in the *General Algebraic Modeling System* (GAMS) environment, the optimal configuration of the electrical power supply system following characteristic restrictions as well as hourly weather and demand data is found. From this model, the optimal mix of solar- and wind-based power generators combined with storage devices and a diesel generator set is formed.

Results: The operation of this model was tested in two real off-grid energy systems, a cluster of villages in India and Titumate in Colombia. Both optimisation processes resulted in hybrid energy systems, utilising photovoltaics (PV), lead-acid batteries and a diesel generator as a load-balancing facility.

Conclusions: With respect to small off-grid energy systems, it was found that renewable energy in combination with electrical storage devices help to reduce the cost of energy compared to stand-alone diesel generator sets. The optimal solutions strongly depend on the particular load demand curve. As both PV and wind energy benefit from energy storage, the costs of the battery can be shared and the two technologies complement each other. Finally, although the optimised capacity of the diesel generator remains nearly constant, its contribution to the total power generation is being substituted by renewable energy sources, which serve as fuel-saving technologies.

Keywords: off-grid, hybrid, optimisation, energy system, linear programming, GAMS

Background

One major issue associated with the challenge of the world's increasing demand for energy is the electrification for approximately two billion people [1] in developing countries that do not currently have access to electricity. The world's population is expected to reach nine billion people in 2050. Most of this expected population growth will take place in developing countries and emerging nations [1]. Additionally, policies in emerging nations are concerned with increasing the supply of energy, as energy consumption per capita has become one of the major indicators for the developmental progress of a country. These two factors mainly determine the growth in energy demand.

Many of the places with limited or no connection to the national grid are rural communities. The question is how to provide these off-grid energy systems. A common solution for off-grid power supply in small and medium-sized energy systems is a fuel generator set [2]; however, the following current developments have sought to improve the competitiveness and desirability of alternative off-grid energy systems:

- Steeply decreasing production costs of renewable energy technologies like solar, wind and biomass caused a boom in the respective technologies in developed countries,
- Expanding research in electric storage devices sparked by the plans of several countries to use electric vehicles in the future,

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- Increasing environmental concerns and awareness of climate change provoked by CO_2 emissions produced by the combustion of fossil fuels and
- Increasing operation costs for fuel generator sets due to rising oil prices.

The following section provides a review of literature on the topic, leading to the presentation of the research questions. In [1], integrated energy farms, which aim to bring power and food to rural communities, are discussed. The supply of electricity is integrated in an independent and decentralised energy supply concept under consideration of sustainable development of remote areas by empowering the residents to take care of their own needs. The study of Nfah et al. [3] is concerned with the optimisation of off-grid energy systems at rural communities in Cameroon focussing on hydro and solar resources. Another more general steady-state modelling approach is done in [4] under consideration of hybrid energy systems consisting of a micro-hydro, a biogas, a biomass, a back-up diesel generator and a photovoltaic (PV) array. The optimal dispatch strategy, as well as the optimal sizing, especially of the PV array is calculated simultaneously by linear programming. The performance of another type of hybrid energy system is investigated in [5]. Here, the interaction between an existing wind/diesel energy system and a lead-acid battery bank is examined. The optimal sizing of such a wind-diesel hybrid energy system is discussed in [6] under consideration of the minimum long-term electricity production cost. In [7], the life cycle costs of stand-alone diesel generator sets and PV battery systems are compared. The benefit from linking different solar home systems by installing an off-grid mini-grid is analysed in [8].

This study seeks to answer the following three research questions:

- Which are suitable elements of an energy system design for off-grid electrification?
- What are the most influential parameters determining the outcome of the optimisation process?
- Which interdependencies among the different technologies in off-grid operation can be found?

In order to find answers to these questions, a general model for a hybrid energy system, consisting of a PV array, a wind turbine, a diesel generator and a lead-acid battery, is developed. First of all, the design, which best interconnects these technologies, is discussed. In modelling, the optimisation takes place with respect to the optimal capacity of the respective technology and simultaneously with respect to the optimal load sharing. This forms a contrast to those models, which have been applied in most of the studies discussed above. Weather and load data of a whole design year are considered within the model[a]. The method of linear programming is applied by using the computer-based modelling program GAMS. The results of the model are displayed by means of real site conditions. A cluster of nine villages named Narendra Nagar in northern India and the village Titumate in the region of Chocó, northern Colombia were chosen as exemplary remote areas for off-grid energy systems. The focus is on finding crucial parameters and interdependencies among the mentioned technologies; therefore, the solution is not so much an optimal proposal for the specific sites. This is because the data sets and assumptions used as the basis for this study are adequate for a theoretical approach. For a real system design, they have to be extended by very specific local parameters and more technologies.

Proposed energy system

The energy system for off-grid remote areas proposed in this study is shown in Figure 1. A three-phase alternating current (AC)-bus-configured grid solution has four major advantages compared to other solutions:

- It is the most easily extendible and modular grid.
- Many of the wind turbines and diesel generator sets are developed for three-phase grid use; large electrical machines will need a three-phase grid.
- The most common electrical devices are designed for AC current, i.e. refrigerators or pumps.
- The frequency of the mini-grid is a valuable signal for:
  - Current load demand and power generation,
  - Generation scheduling, a suitable dispatch strategy and
  - Demand side management[b].

Power generation options

The three power generation units considered in this paper are PV, a wind turbine and a diesel generator set. Unlike micro-hydro, natural gas, biogas, biomass and other possibilities for off-grid power generation, the substantial problem of temporal inequalities of power supply and demand is attached to wind and solar energy. This is why their interaction with a diesel generator set as the most common and controllable off-grid solution in combination with an energy storage device has to be examined in detail when analysing their suitability for off-grid energy supply. As mentioned above, this study does not aim to find the entirely optimal design proposal for the specific sites because in this case, more technologies should be taken into consideration.

Photovoltaic

Global production of PV systems has been increasing substantially in the past years. The main driver was a heavy cost decrease: While the initial cost of PV modules

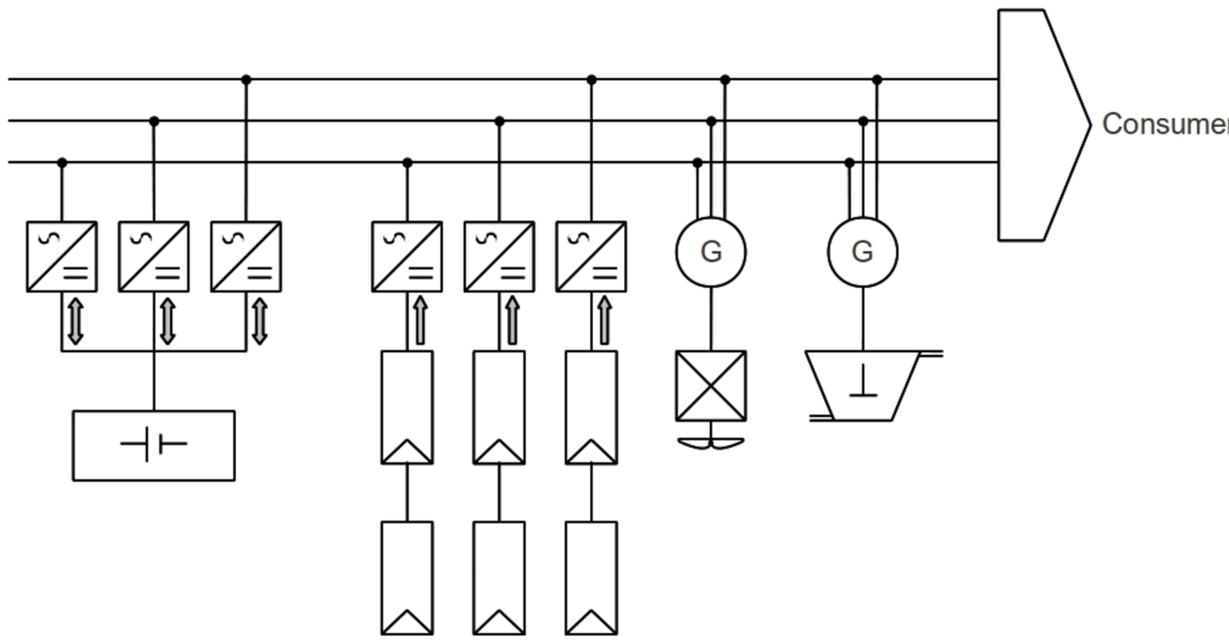

Figure 1 Remote area energy system designed.

of up to 100 kW_p was €5,000/kW_p[c] in the second quarter of 2006, the cost sunk to €2,912/kW_p in the second quarter of 2010 [9].

Due to economies of scale, the cost of energy in a small off-grid energy system is generally higher than that in an on-grid system. Thus, PV reaches cost competitiveness first in off-grid solutions. According to [7], the life cycle unit cost of PV energy in India was cheaper or comparable to that of a diesel generator in 1999 at a diesel price of €0.31/l. PV is modular in nature and consequently easy to scale and expand. These are two essential qualities for off-grid utilisation. A very common solution for the off-grid use of PV are solar home systems (SHS). They consist of a PV array and a battery and meet only basic electric demands as they use direct current[d].

The energy system introduced in Figure 1 has, in contrast to a SHS, an AC-bus configuration. Because PV panels provide DC, inverters are essential for the implementation of PV into the energy system. The inverters reduce the efficiency of PV by about 5% from 11-12% to 10.5-11.4% [3,4].

Wind power

Like solar energy, wind energy is experiencing a profound reduction in production costs. Until 2010, there was a trend in the wind energy market of upscaling wind turbine capacities[e]. This is in contrast to PV technology, which is modular in nature and thus without such a trend. The specific investment costs for small-scale wind turbines (€5,832.00/kW[f]) were about four times higher than the average wind power installation costs in Germany in 2009. In 2008, 38.7 MW of small-scale wind turbines were installed worldwide - an increase of 53% over 2007 [10]. The market for small-scale wind turbines below 100 kW_r is comparably small in Germany.

Small wind turbines play an important role in decentralised energy systems. As [6] shows, wind-diesel hybrid energy systems can reduce costs of energy drastically compared to diesel-only solutions. The two sites in Greece presented in that study with average wind speeds of 10 and 6 m/s show cost reductions of 60% and 20%, respectively, over 20 years. Thirty islands independent of the respective national grid are presented as examples in [11], many of them are working with small wind turbines. The economic benefit from these solutions is emphasized in comparison to diesel generator sets. Wind-diesel hybrid energy systems for islands are also considered in [12], whereby a minimum average wind velocity of 5 m/s is proposed.

However, [10] as well as [5] indicate that small-scale wind turbines do not show an entirely good technical performance. This becomes obvious when comparing the power output curve given by the manufacturer to the measured real output in [10] (page 23), which displays a considerable smaller real capacity. Additionally, within this study, it is emphasized that horizontal small-scale wind turbines generally perform better than vertical turbines. The predicted annual production of electric power of 30,000 kWh compared to the actual field data of 12,355 kWh in [5] (page 435) for a wind turbine of 10 kW_r at an average wind speed of 7.9 m/s provides an additional example of the mentioned performance problems.

Diesel generator set

Due to the low initial costs, diesel generator sets remain the most commonly used technology for the electrification of people in remote areas, *cf.* [2]. The following reasons account for the attraction of diesel generators for off-grid use:

- The suitability for hybrid energy systems through its controllability,
- The worldwide availability of diesel fuel and
- The widespread and typically easy-to-maintain technology.

High-performance diesel generator sets are designed to feed a three-phase grid. Therefore, a three-phase configuration is recommended for an off-grid energy system. The diesel generator's crucial parameter is its efficiency, which is measured as the full-load efficiency. In hybrid off-grid energy systems configured as proposed above, the diesel generator is generally the only controllable device. It has to track the load and must be able to cover peak load. Therefore, its amount of hours of full-load service per year is comparably low. In contrast, the diesel generator often runs:

- At partial load,
- For short time periods and
- With a high number of start-stop cycles.

This is especially true for hybrid systems, where other fluctuating energy generation options lead to a less predictable residual load curve with even higher variations. Thus, the average coefficient of performance is lower than the rated efficiency, and comparably high operation and maintenance costs are observed. The overall performance of diesel generators in off-grid energy systems is lower than that in on-grid operation.

Battery

The key to facing the fundamental problems of temporal inequalities of solar or wind power and electric power demand is an energy storage device. Today, the known crucial technical solutions[g] in this context are:

1. Mechanical energy storages:
   - pump storage hydropower plant,
   - compressed air store and
   - flywheel and
2. Electrochemical energy storages:
   - lead-acid, nickel metal hydride and lithium-ion cells,
   - high-temperature traction batteries,
   - fuel cells and
   - flow batteries.

The lowest capacity cost, the reliability and (with about 3,000 to 5,000 numbers of charging-discharging cycles over life) the comparatively long durability of lead-acid batteries are the three main reasons, why today this is the default solution for off-grid energy storage. Also, they can be discharged fast, which means that they can serve peak load for a short time[h]. Their energy density is 20 to 30 Wh/kg, which is relatively small. Lead-acid batteries show a self-drain of 3% to 5%/month, which is caused by the internal resistance of the battery. The efficiency can be estimated at about $\frac{E_{\text{removed}}}{E_{\text{input}}} = 85\%$. The performance of lead-acid batteries suffers at temperatures below 0°C and above 60°C [13] (pages 33-61). The depth of discharge is limited to 50% to 80% of the overall battery capacity; the depth of discharge represents the fraction of the battery that can be used without damaging the battery through internal irreversible processes, which shorten its capacity and durability. There are two other energy storage technologies that should be considered in the context of off-grid energy systems:

1. According to [12], the flywheel performs well in interaction with wind energy, as it perfectly absorbs peak loads aroused by short but high wind velocities and serves peak load demands.
2. High-temperature traction batteries have the potential to serve as centralised storage devices for medium-sized and large off-grid systems[i].

In order to reduce the complexity of this energy system model, only lead-acid batteries are applied because the simulation of the self-discharge of the flywheel cannot be realised within the scope of this study and because there are no viable economic data for the high-temperature traction battery yet.

Methods

As mentioned above, the optimisation process within this study is implemented using linear programming. This method allows only linear dependencies between all occurrent decision variables. These dependencies are the model's characteristic restrictions limiting the optimal solution of the objective function. The following sections provide more detailed information on the model's objective function, decision variables and restrictions.

Assumptions

In modelling the technologies, the economic performance and the site data are the crucial components to optimising an off-grid energy system. The simultaneous optimisation of the utilisation and of the capacities of the considered technologies account for the following assumptions:

1. *Steady state of the system and all technical and economic parameters*: On the economic side, this approach mainly is not unerring for diesel fuel[j]. Changing prices influence their competitiveness. The influence of diesel price variations is examined in detail in Figures 2 and 3.
With respect to the steady state of technical parameters, the output of the PV array and the wind turbine is supposed to depend only on meteorological data linearly (and not on the facility's age). Additionally, a constant efficiency of 40% for the diesel generator and of 90% and 95% for the battery charging or discharging, respectively, is assumed. The battery's capacity is assumed to be constant over its lifetime. The first two assumptions are realistic, as PV's and wind turbine's long-term loss of efficiency is not significant. With the assumptions concerning the battery and the diesel generator, the simulation can run for 8,760 h (exemplary for the whole lifetime), and the values therefore must be regarded as averaged values. The viability of these assumptions for the diesel generator and the battery is discussed below.
2. *Weather and load data are constant within each 1-h time step*: The dependency of PV and wind energy on fluctuating weather conditions on one side and the inconstant load profile on the other account for the necessity of a time-dependent simulation. As a trade-off between constraints regarding the availability of highly time-resolved data and a realistic and detailed simulation, 1-h time steps are used within this study.
3. *The available load data are representative for the whole economic lifetime*: This assumption is more of a data constraint than a willingly chosen assumption, as

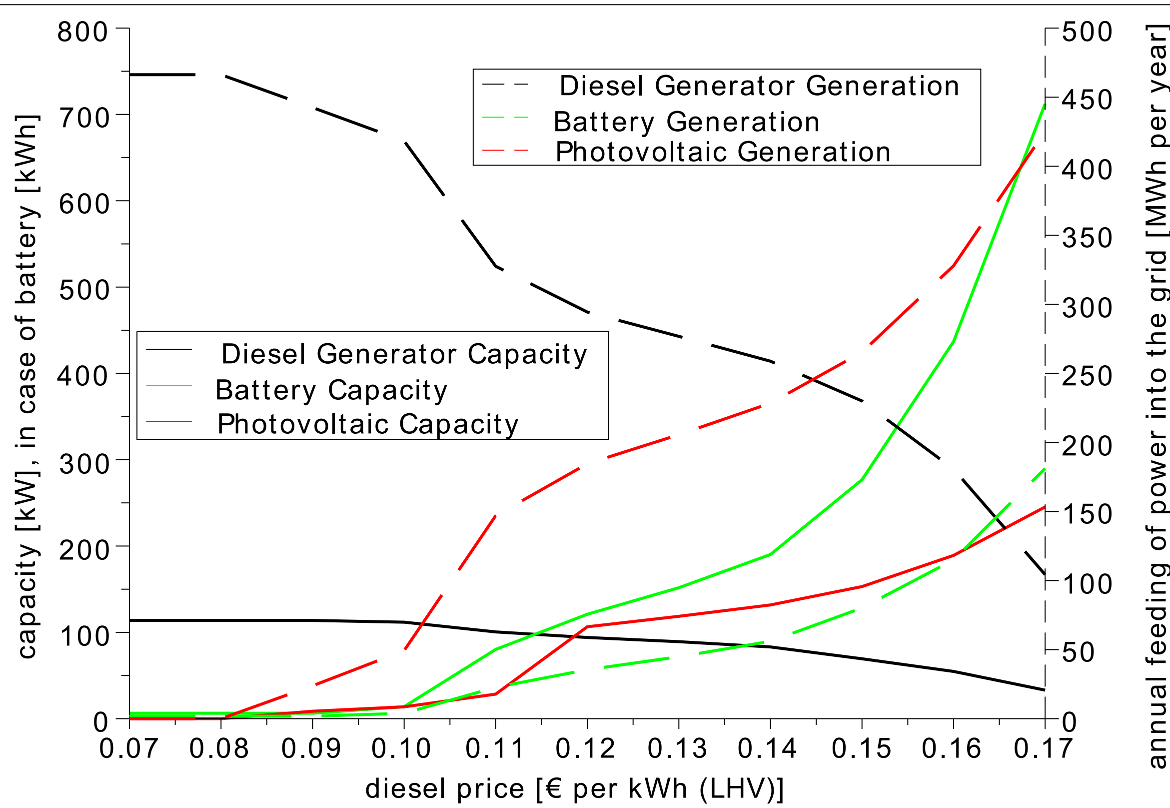

Figure 2 Influence of increasing diesel price on optimal solution, Indian case study.

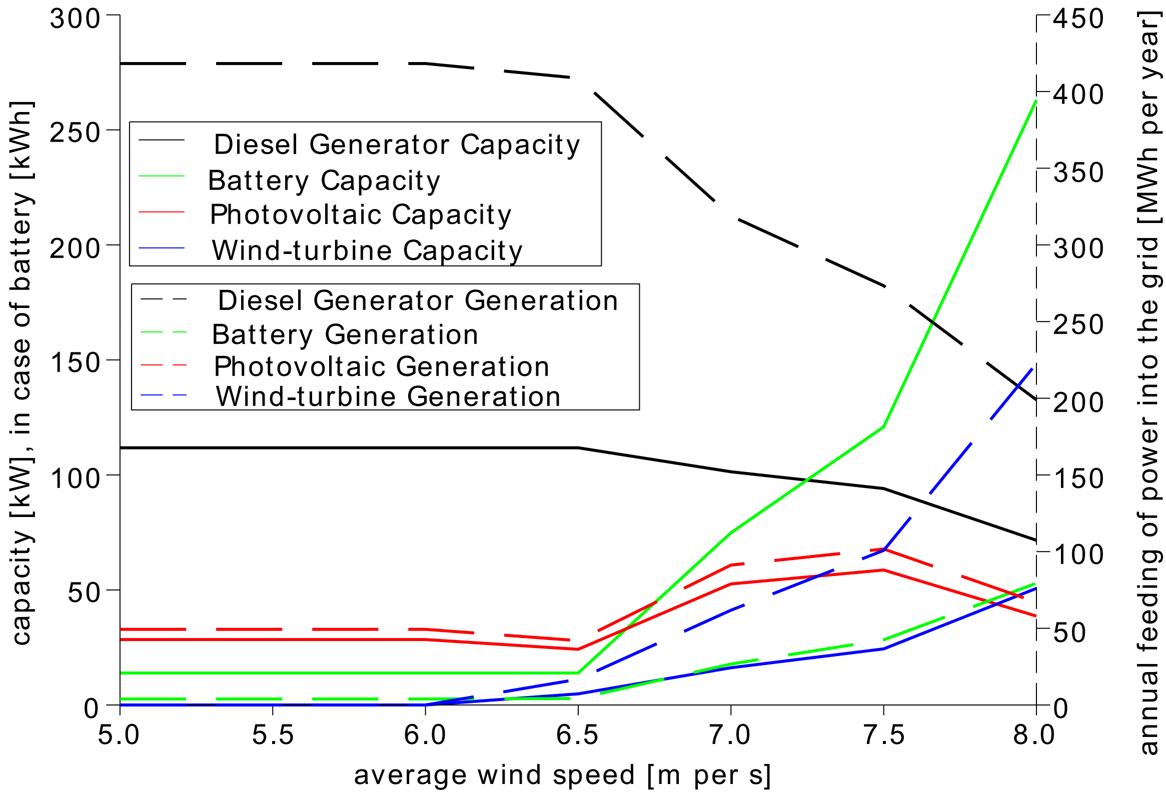

Figure 3 Influence of increasing diesel price on optimal solution, Colombian case study.

for most remote areas, no empiric annual load profile is available. Long-term, seasonal and even weekly changes are neglected[k].

4. *The applied weather data of a design year and in parts also of a design region are representative*: In order to obtain a reasonable solution, it is of utmost importance that the applied weather data contain seasonal changes and contribute equally to a balanced complexity of the whole model. Thus, one weather data set of a design year is the basis for the output of the PV modules and the wind turbine. If no weather data is available for a particular region[l], it has to be taken from a design region and adjusted to all known local conditions.

5. *Maintenance and operation costs depend only on the installed capacity*: The maintenance and operation costs of the PV system, the wind turbine and the battery do not correlate linearly with their usage; in comparison to the dependency on their capacity, this correlation can be neglected. For the operation and maintenance costs of the diesel generator (of course, the diesel consumption is considered separately), there are interdependencies between usage, start-stop cycles and the amount of hours running on full or partial load. As there is no reliable database or function accessible, which could assist in finding a linear estimation, these correlations are not integrated in the model.

6. *Maintenance and breakdown interruptions are not considered*: The possibility of power blackouts in an off-grid energy system is relatively high, as there are only a few devices to compensate for the breakdown of another one. Nevertheless, power blackouts have not been regarded. Power is therefore assumed to be supplied without interruption. The vulnerability of an off-grid energy system has to be regarded separately from this optimisation.

7. *The total PV electricity generation is estimated by applying full-load hours while only the global horizontal solar radiation has an influence on the hourly PV power output*: The full-load hours guarantee a reasonable total annual PV output, and the global horizontal radiation allocates the PV output hourly. Moreover, it could be discussed if radiation on an inclined surface led to other solutions than assuming horizontal radiation. Usually, when designing a PV array, this is done during the calculation of the electricity yield. As the PV output is already estimated with full-load hours, this is not necessary (although it would be even more precise to do so).

8. *The power generation of the wind turbine depends on the actual wind velocity by means of a rated wind speed-power curve*: Generally, the output of the wind turbine depends on the wind speed, the air density and the efficiency. These dependencies are broken down to a load curve. Thereby, the effect of the air density is neglected.
9. *There is no self-discharge of the lead-acid battery*: Because the average duration of energy storage in the lead-acid battery is shorter than 1 day, the self-discharge of up to 5% (*cf.* [13] (pages 33-61)) per month is neglected.
10. *A technology for dump load and its cost are not considered in this model:* The modelled energy system is allowed to dump surplus energy for free. Especially at night - when there is nearly no load demand - it occurs that high wind velocities bring a lot of surplus electricity into the grid, which can be used to feed the battery. When the battery is charged completely, a dump load has to absorb the surplus energy in order to avoid a blackout as a consequence of a rising frequency. Alternatively, the wind turbine and the control system are designed to automatically turn it off.

Model formulae

The formulae for this specific hybrid energy system model are presented in the following paragraphs. The objective function below sums the annualised specific capital costs as well as operation and maintenance costs (all symbols are given in Table 1).

As mentioned above, the modelling horizon is 1 year, and therefore, all costs (tc, mc_j and cc_j) are annualised and are capacity-specific values. The CRF^{m} is used to calculate these costs. The calculation is presented in detail in Table 2. Both, the capacity and the operation of each technology, are decision variables; the energy system is optimised considering both variables. The minimisation of tc in Equation 1 is performed under the capacity restrictions (valid for all t)[n]

$$tc = \sum_{\text{j}} c_j \cdot (mc_j + cc_j) + \frac{p_d}{\eta_{dg}} \cdot \sum_{t=1}^{8,760} x_{dg,t} \tag{1}$$

$$x_{j,t} \leq c_j \cdot cf_{j,t} \tag{2}$$

$$x_{bc,t} \leq \frac{c_{bc} - bs_t}{\eta_{bc}} \tag{3}$$

Table 1 List of symbols

Symbol	Description	Unit
j: dg	Dieselgenerator	
pv	Photovoltaic	
w	Wind turbine	
bc	Battery charging	
bd	Battery discharging	
η_j	Efficiency of technology *j*	-
$S_{g,t}$	Global horizontal radiation at time segment *t*	Wh/m^2
cc_j	Annualised specific first cost of technology *j*	€/kW
$P_{e,off}$ $P_{e,on}$	Off-grid and on-grid electricity price	€/kWh
c_j	Capacity of technology *j*	kW
n_{inv}	Efficiency of inverter	-
FLH	Annual full-load hours of PV	h/a
mc_j	Specific maintenance cost of technology *j*	€/kW
TPC	Total annual power consumption	kWh
$x_{j,t}$	Usage of technology *j* at time segment *t*	kW
tc	Total annual cost of energy system	€
v_w	Wind velocity	m/s
p_d	Diesel price	€/kWh
CGE	Specific cost of grid extension to national power line	€/km·a
$cf_{j,t}$	Capacity factor of technology *j* at time segment *t*	-
bs_t	Battery charging status at time segment *t*	kWh
d	Break-even distance to national grid	km

$$x_{bd,t} \leq bs_t - c_{bd} \cdot (1 - cf_{bd,t}) \quad (4)$$

and the demand restriction, given in Equation 5:

$$ld_t \leq x_{dg,t} + x_{w,t} + x_{pv,t} \cdot \eta_{inv} + x_{bd,t} \cdot \eta_{bd} - x_{bc,t} \quad \text{for all } t. \quad (5)$$

The additional capacity restrictions for the battery (given in Equations 3 and 4) are necessary because the battery status, which varies over time, determines the available battery capacity. $x_{j,t}$, c_j and tc are set to variables of a positive value, so the optimisation process provides only reasonable solutions.

Technologies

While in the former section the model's general formula are presented, the focus in this section lies on the characteristics of the technologies used in this model. The capacity factor $cf_{j,t}$, which appears in the capacity restrictions (Equations 2, 3 and 4), indicates what fraction of the installed capacity is available at time segment t. The capacity factor for the diesel generator set is 1°. Furthermore, the capacity factor for PV depends on the solar radiation and for the wind turbine on the wind velocity.

Photovoltaic

Usually, the annual output of a PV array is calculated by means of the full-load hours at a specific site. In order to optimise the dispatch strategy, information about the moment of energy generation is essential. Thus, the global horizontal solar radiation[p] is needed for every single hour of the design year's 8,760 h in a reference region. The PV capacity factor is calculated as follows:

$$cf_{pv,t} = \frac{S_{g,t} \cdot FLH}{1h \cdot \sum_{t=1}^{8,760} S_{g,t}}. \quad (6)$$

Consequently, as the radiation data at the specific site usually are not available, the required site data consist of the expected full-load hours and the hourly global solar radiation of a reference region. The assumption concerning PV mentioned above primarily neglects the influence of the temperature on the efficiency of PV modules. This will cause errors at sites that experience high seasonal variations in temperature.

Wind turbine

The output of a wind turbine depends primarily on the wind velocity and the air density. While the kinetic energy of wind increases proportionally to the cube of the wind velocity, this is not the case for the wind turbine's output at every operating point. For this reason, the wind turbine's load curve is estimated by means of an averaged manufacturer's power output curve instead of the wind's kinetic energy and the efficiency. The cut-in and furling wind speed are set to 3 and 20 m/s, respectively, and the load curve is estimated as follows within this range [10] (page 23):

$$cf_{w,t} = 0.0075 \cdot 1.6^{v_w} \quad \text{for } 3\,\text{m/s} \leq v_w < 10\,\text{m/s} \quad (7)$$

$$cf_{w,t} = -0.05 + 0.0875 \cdot v_w \quad \text{for } 10\,\text{m/s} \leq v_w < 12\,\text{m/s} \quad (8)$$

$$cf_{w,t} = 1 \quad \text{for } 12\,\text{m/s} \leq v_w \leq 20\,\text{m/s}. \quad (9)$$

In order to estimate the mean wind speed, the required site data are the annual mean wind speed at the site at a height of 30 to 50 m and the hourly wind speed at a reference region, if this wind data is not available at the precise site. The hourly wind speed is estimated by

$$v_{w,t,s} = \frac{av_{w,s} \cdot v_{w,t,r}}{av_{w,r}}, \quad (10)$$

where v is the wind speed at time segment t, and av is the averaged wind speed at the site or reference region.

Table 2 Economic parameters of all considered technologies

Technology	Unit	Photovoltaic	Wind turbine	Diesel generator	Lead-acid battery
Investment cost	€/kW (€/kWh for battery)	2,835.00	5,832.00	596.00	148.00
Lifespan	years	20	20	20	5
Interest rate	%	10	10	10	10
CRF	-	0.1175	0.1175	0.1175	0.2638
Annualised	€/kW	333.00	685.02	70.01	39.04
Investment cost	(€/kWh for battery)				
Maintenance and operation cost	% of investment cost	2	2	6.4	2
Maintenance and operation cost	€/kW (€/kWh for battery)	56.70	116.64	38.08	2.96
STMGC	€/kWh_{electr}	0	0	0.25 (India), 0.18 (Colombia)	0

Adapted from [14] (diesel generator), [9] (PV), [10] (wind turbine) and [15] (battery).

Diesel generator

As mentioned above, the capacity factor for the diesel generator is set to 1 for every time segment during the year. Hence, in Equation 2, the maximum capacity of the diesel generator is its rated capacity. As assumption 6 above indicates, maintenance and breakdown interruptions are not considered hereby. In addition to that, the constant efficiency of 40% (*cf.* assumption 1) allows the diesel generator to follow the load profile - more precisely, the gap between the load profile and the renewable energy supply - without further constraints.

Lead-acid battery

As Equations 3 and 4 show, the usage of the battery depends on the capacity factor and the battery status bs_t. The capacity factor represents the fraction of a technology's capacity that can be used at time segment t; in the case of the battery, this is the depth of discharge: $cf_{bd,t} = cf_{bc,t} = 0.8$. Equation 11 shows how the battery status is calculated depending on its charging or discharging operation mode.

$$\begin{aligned} bs_{t+1} &= bs_t + x_{bc,t} \cdot \eta_{bc} - x_{bd,t} \\ \text{initial value:} \quad bs_1 &= (1 - cf_{bd,1}) \cdot c_{bd} \end{aligned} \tag{11}$$

The charging and discharging of the battery has been split up into two variables in order to be able to simultaneously separate the capacity restrictions in Equations 3 and 4 and to apply the different efficiencies for each operation mode. Charging efficiency is set to 90%, and discharging efficiency, to 95%, which leads to a round trip efficiency of 85.5%. Hereby, the total investment costs of the battery (the calculation and magnitude of the economic parameters are given in the next section and in Table 2) are allocated to the charging capacity variable c_{bc}; thus, the modelled capacity of the discharging battery c_{bd} is free of cost.[q]

Economic parameters

In this section, the values of the economic parameters are presented. In order to include the whole life cycle into the cost consideration, the annuity method is applied. All economic parameters are summarized in Table 2. The costs are specific net investment costs, including acquisition and installation costs. One reason for the necessity of the annuity method is the different economic life span of the lead-acid battery (5 years) and the other technologies (20 years). As the annuity method allocates the investment and interest costs annually over the respective technology's life span, the differences between the life spans are considered. The diesel fuel price is calculated by considering the lower heating value.[r] As [7] (page 318)shows, the different rates of interest have a vast influence on the competitiveness of renewable energy generation units with their comparably high investment costs. Following World Bank guidance, the rate of interest is set to 10%, while in [3] (page 1069) and in [6] (page 1394), the rate of interest or mean annual capital costs are estimated to be 6% and 8%, respectively. If specific subsidies are not considered, there is no reason why in rural areas, capital costs should be disproportionately low.

The specific cost of energy $\mathrm{COE} = \frac{tc}{\sum_{t=1}^{8,760} ld_t}$ allocates the total cost of the energy system to the annual generated electricity. Generally, technology-specific costs can be split up into short-term marginal generation costs (STMGC) and long-term marginal generation costs (LTMGC). STMGC include all costs that depend on the operation of a technology, whereas LTMGC additionally include all costs not depending on its operation.

$$\mathrm{STMGC}_j = \frac{p_j}{\eta_j}, \tag{12}$$

where p_j is the price of the energy resource consumed by j. According to assumption 5 above, maintenance and operation costs depend only on the installed capacity. Consequently, these costs only appear in the LTMGC. Equation 12 is only important for the diesel generator, as solar radiation and wind energy are free of charge. This is the reason why they are used when available. The term $\frac{\sum_{t=1}^{8,760} x_{j,t}}{c_j \cdot 1a}$ is equal to the technology's full-load hours.

The calculation of the break-even grid distance is given in Equation 13:

$$d = \frac{(\mathrm{P_{e,off}} - \mathrm{P_{e,on}}) \cdot \mathrm{TPC}}{\mathrm{CGE}}. \tag{13}$$

The break-even grid distance gives the particular distance where extending the grid to the national power line shows the same economic performance as building up an independent off-grid energy system. Consequently, it gives the minimal distance to the national grid, which makes the off-grid energy system economically viable. The grid costs are regarded as external costs because for both possibilities (grid extension or independent solutions), the same costs arise while the specific focus of this study lies on cost differences and not on absolute values. The off-grid electricity price is estimated with the optimised cost of electricity generation, whereas the actual average electricity price of the national electricity suppliers represents the on-grid electricity price. The specific costs of a grid extension are estimated to be €864.92/km·a, *cf.* [14] (page B-5).

Results and discussion

In the previous sections, general assumptions and the modelling were discussed. The model is applied to reflect real site conditions in the following sections. As mentioned above, the case studies were conducted using data from Narendra Nagar, India and Titutmate, Colombia.

Case study I: Narendra Nagar, India

The Narendra Nagar block of the district Tehri Garhwal, Uttarakhand State in northern India consists of nine villages without access to the national power line. It is a mainly hilly, remote area, and electrification by grid extension is considered economically unviable [4] (page 528).

In addition to the economic data given in Table 2, the site conditions are estimated with the parameters in Table 3. The load curve, which was applied for this area, is displayed as the pink, continuous line in Figure 4. As there were no weekly load profiles accessible, the given daily profile is regarded as typical and covers all 365 days in the simulated year. Especially on weekends and holidays and also induced by climate changes over the year, the real load profile moves away from the given. Nevertheless, it works well in the simulation context, as it bears all the issues connected to an off-grid energy system:

- Nearly no demand for electricity in the night,
- A morning peak with up to 69 kW from 8:00 a.m.,
- A peak of 111 kW at 1:00 p.m. and
- A long-lasting evening peak from 6:00 to 11:00 p.m. going up to 114 kW at 9:00 p.m.

According to [4] (page 531), the three biggest consumer devices in this energy system are lighting, fans and televisions. In the winter period, fans are not used to the same extent as in summer; thus, the load profile is taken from the summer period in order to enable the energy system to deal with peak loads in the summertime.

The results of the general optimisation are displayed in Table 4. In Figures 2, 5, 6 and 7, the sensitivity analysis is presented. For every figure, only one parameter is varied, all other parameters are constant and of the same value as in the general optimisation. Thereby, the optimisation process is executed several times. Accordingly, each point in the graphs shows an optimal solution for the respective varied parameter. Figure 5 demonstrates the results for the variation of the average wind speed, and Figure 2 shows the effects of varying the diesel price. The influence of the battery's and PV system's costs on the optimal solution is examined in Figures 6 and 7, respectively. Figure 4 shows the contribution of each technology in serving the load demand for a design day and an elevated average wind speed of 7.5 m/s. The dispatch strategy of the respective technology and the interaction of all four technologies can be determined from this figure. The elevated wind speed is chosen in order to include conclusions for wind energy in off-grid energy systems and to find interdependencies between wind energy and the other technologies.

As the results in Table 4 indicate, the optimal energy system mainly consists of a diesel generator assisted by PV and a small lead-acid battery. The optimal capacity of the generator is equal to the peak load demand. Wind energy is not viable for the given wind velocities. For an average wind speed above 6 m/s, wind energy does play a role, *cf.* Figure 5. Starting with this average wind speed, both PV and wind turbine capacities as well as their generation start to increase. From 7.5 m/s on PV capacity decreases again, while the wind turbine capacity keeps increasing. Additionally, the capacity of the battery rises with an increasing amount of renewable and fluctuating energy. Even for good wind conditions, the wind turbine's capacity remains below 50 kW.

As Figure 2 shows, an elevated diesel price increases the contribution of PV to the energy system. By doubling the diesel price to €1.4/kWh, the optimal fraction of PV power generation rises from 0% to 50%, whereas the capacity of the diesel generator diminishes very slowly. The usage of the battery follows the rising PV capacity proportionally, while its capacity increases disproportionately high. Interestingly, a rising diesel price does not lead to an inclusion of wind energy in the system.

In Figures 6 and 7, cost reductions of PV and the lead-acid battery (given in percent) result in a higher fraction of PV energy and capacity and simultaneously in an increasing inclusion of energy storage in the energy system. In both cases, wind energy is not included in the optimal design of this energy system. The system has a greater sensitivity to cost reductions in PV than to decreases in battery costs. As for the variation of the diesel price, the battery capacity increases more steeply than its utilisation. In particular, the solution for decreasing battery costs is characterised by this behaviour.

Table 3 Site data

	Unit	Data for India	Data for Colombia
Average wind speed	m/s	5.2	3.1
PV full-load hours	kWh/ kW_p·a	1,825	2,100 [16]
Diesel price	€/kWh	0.1	0.072
Reference hourly global radiation	Wh/m^2	*cf.* [17][a]	*cf.* [17][b]
Reference hourly wind speed	m/s	*cf.* [17][a]	
Peak load demand	kW	113.8	46.76
Annual power consumption	MWh	466	222

[a]Dehradun, Uttar Pradesh, India; [b]Bogotá, Colombia.

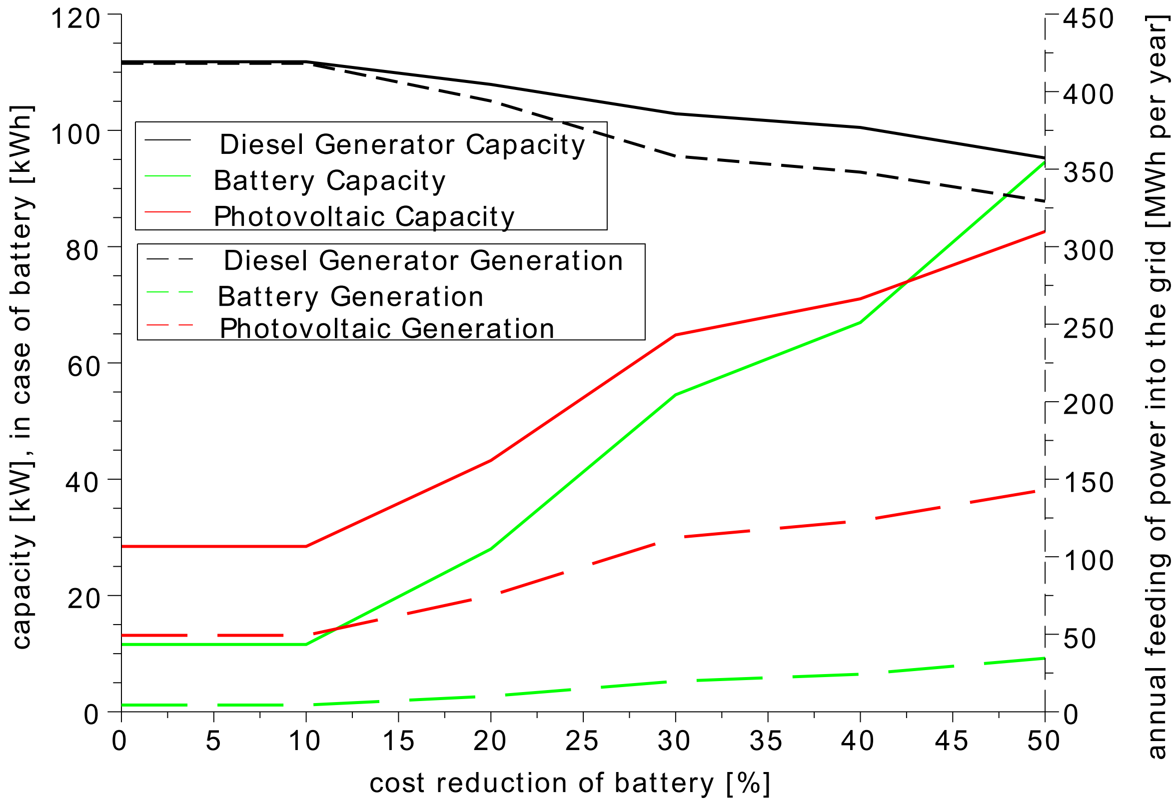

Figure 4 **Exemplary optimal power generation distribution strategy for 1 day with solution for an elevated wind speed of 7.5 m/s**.

The calculated off-grid cost of electricity of €27.5 ct/kWh lies within the range of €15.07 to €40.13 ct/kWh which are estimated costs for the diesel generator stand-alone solution given in [14] (page 55). It is more expensive than the solution proposed in [4] of €10 ct/kWh, which includes more technologies: a micro-hydro, biomass and biogas unit. The break-even grid distance of 113 km is a strong restriction for an off-grid energy system, although the assumed specific cost of a grid-extension of €864.92/km·a [14] (page B-5) is averaged over normal terrain in India and therefore is higher at hilly sites like Narendra Nagar. The reference electricity price of €6.5 ct/kWh is small[s].

Figure 5 reveals three dependencies:

Table 4 Optimal solution parameters for the first case study (India)

Parameters	c_{dg}	c_{bat}	c_{pv}	c_{cw}	d	Cost of electricity generation
Unit	kW	kWh	kW	kW	km[a]	€ct/kWh
Value	113.8	11.6	28.4	0	113	27.5

[a]Break-even distance to the national grid, applying €864.92/km·a for a national power line grid extension and an electricity price of €6.5 ct/kWh.

1. The higher the average wind speed, the more wind energy is used. This confirms the good performance of wind-diesel hybrid energy systems discussed in [5,6] for average wind speeds of 7.5 m/s.
2. The diesel generator set is still at high capacity to serve peak loads, but its annual contribution to the power supply is diminished disproportional to the capacity. Also, when increasing the competitiveness of the battery, the PV array or the diesel price, a slowly diminishing diesel generator capacity is found.
3. The increasing battery capacity caused by high wind energy generation is an advantage for PV as well. Its optimal capacity nearly doubles from 28.44 to 58.66 kW. Nevertheless, an ongoing improvement of wind conditions leads to a decreasing optimal PV capacity.

Based on these results, wind energy and PV can positively interact in off-grid energy systems under certain circumstances. One reason for this conclusion is that both technologies depend on the lead-acid battery bank, which they can use separately to some extent. The decreasing optimal PV capacity for average wind speeds above 7.5 m/s shows that this synergy is only possible within specific limits; beyond these limits, the two

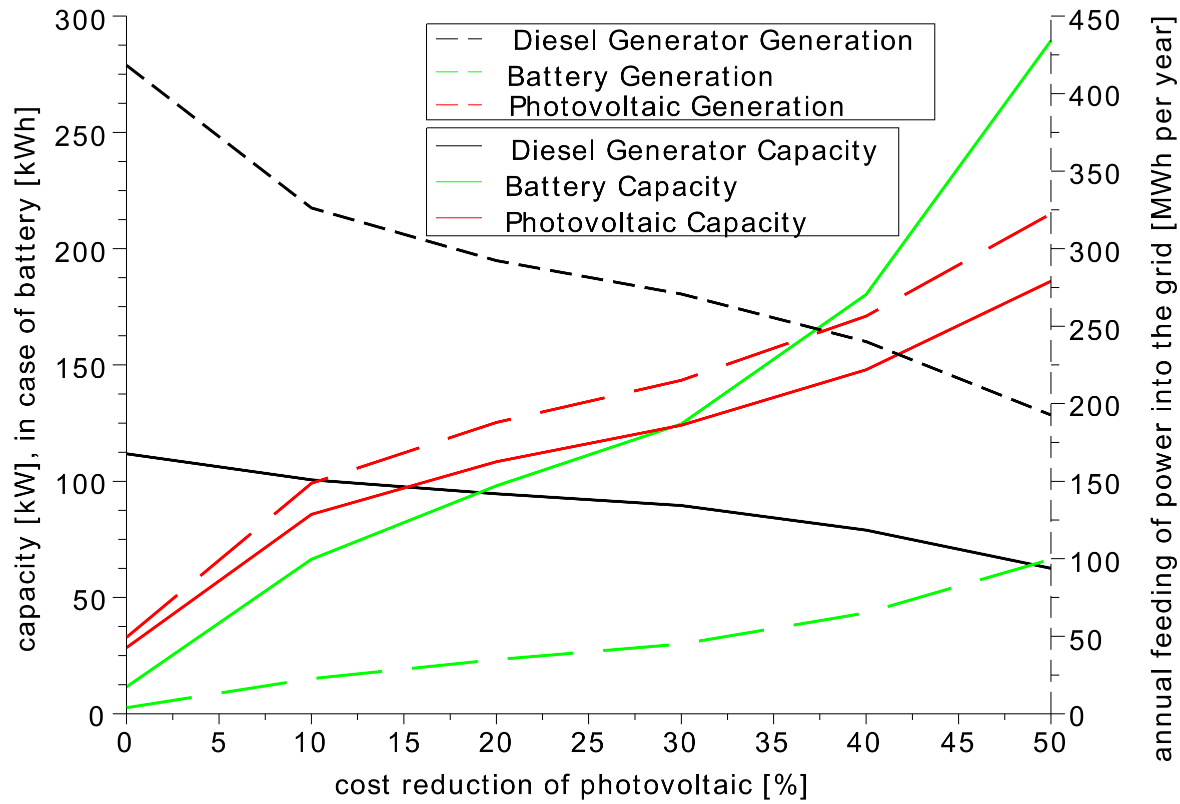

Figure 5 Influence of average wind speed on optimal solution.

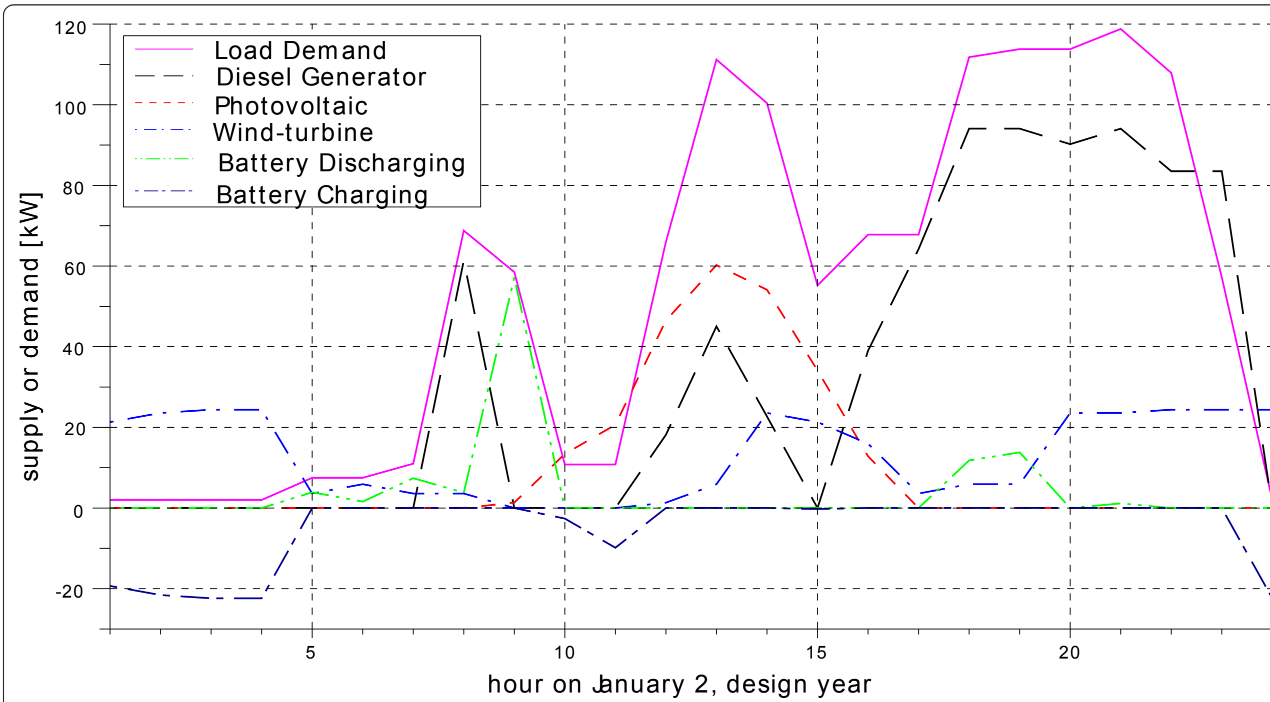

Figure 6 Influence of battery cost reduction on optimal solution.

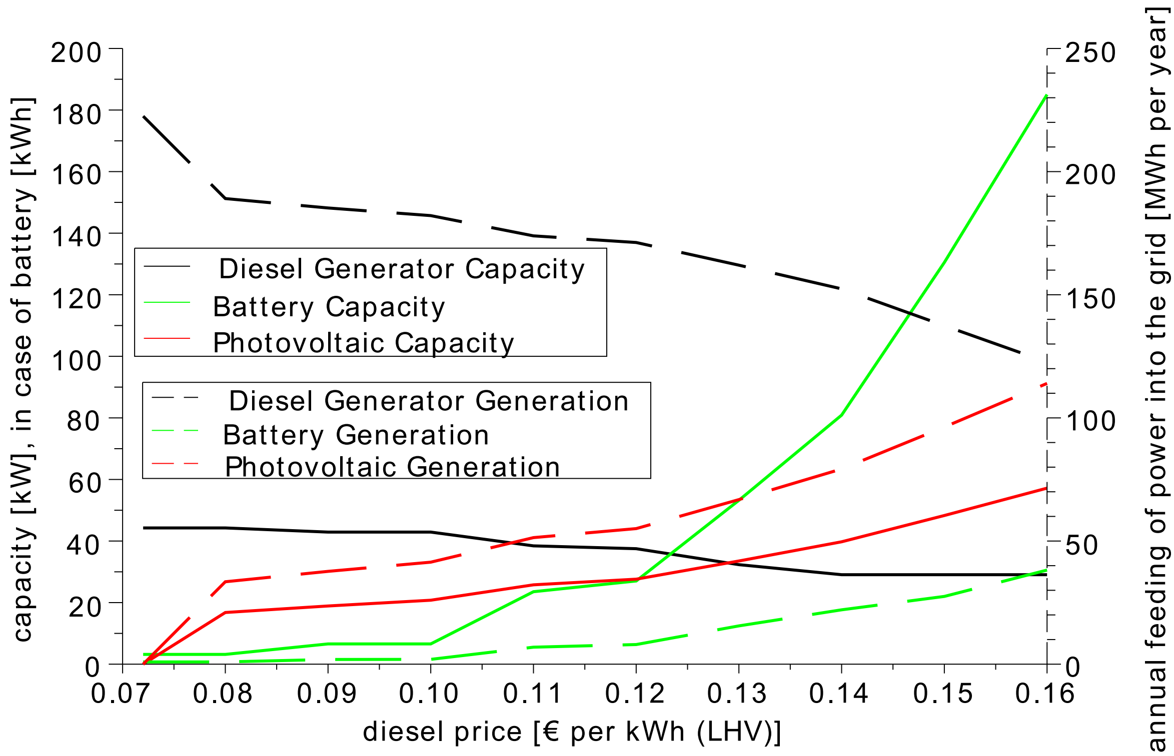

Figure 7 **Influence of PV cost reduction on optimal solution**.

technologies compete. Although for reasons of security of energy supply, more technologies are an advantage, from an economic point of view focussing on the technology with lower long-term generation costs is advisable outside the synergistic area. This relation between the two technologies is economic and not technical, as the efficiency and power generation of the technologies are not correlated. The battery can be seen as an essential upgrade for both technologies, which is necessary for solving the problem of temporal inequalities of power supply and demand. The cost of this improvement can be shared by the two technologies.

The vast diesel price sensitivity of the energy system is shown in Figure 2. Doubling the diesel price completely turns the energy system upside down, and the diesel generator is used as a peak-shaving facility only. The demand for battery capacity grows disproportionately to that of PV, and the cost of electricity rises as well. If the diesel price increases to €12 ct/kWh, electricity costs grow to €31.1 ct/kWh. For a diesel price of €17 ct/kWh, electricity costs increase to €37.2 ct/kWh. As in the general solution, no wind energy is included in the results for increasing diesel prices. This is an effect of the model's preference for the technology with lower long-term generation costs. If the difference between the economic competitiveness of PV and of the wind turbine is too large, there is no positive interaction between the two technologies. The disproportionally increasing battery capacity shows that with a decreasing diesel generator capacity, there are situations when an extra capacity is needed. This capacity is supplied by the battery when PV electricity is not available. As these situations occur only rarely, this extra capacity stays unused in other respects. On the one hand, this increases the unit cost of the usage of the battery, but on the other hand, it provides the possibility of diminishing the number of cycles over life and depth of discharge of the lead-acid battery as the grid manager has an increased space for controlling the charging or discharging behaviour.

Although optimal solutions excluding the lead-acid battery bank show a minimised capacity of the PV system and of the wind turbine, the energy system is not very sensitive regarding price variations of the battery, which can be seen in Figure 6. One reason for this is that a direct correlation between the number of life cycles and the overall lifetime of the battery has not been included in the model. Consequently, a greater utilisation of the battery (as an effect of an increased PV or wind turbine capacity) does not affect its economic lifetime. This model-inherent advantage for the battery reduces the constraint, which the battery bank's cost represents for the energy system. Nevertheless, the comparably small sensitivity of the energy system leads to the conclusion that the potential of off-grid energy systems including PV and wind turbines is mainly restricted by their costs. This is shown in Figure 7. Here, the share of the generated power of the diesel generator and PV system, respectively, are equal, if costs are reduced by 37%, which is equal to a PV price of €1,787.31/kW$_p$. Taking into consideration the decreasing production costs of PV (*cf.* [9]) and the increasing cost for diesel fuel, there is a strong economic motivation for the usage of PV in off-grid energy systems. The substitution of diesel generators takes place as a substitution of generated energy and not to the same extent as a substitution of the diesel generator's capacity. This conclusion has already been discussed above.

On the one hand, the dispatch strategy that leads to Figure 4 is optimal concerning the utilisation of the battery. There is wind power, but no significant demand for electricity at night and the battery is charged. From 10:00 to 12:00 a.m., the PV power generation is higher than the demand for electricity, and again, the surplus energy can be stored in the lead-acid battery bank. This surplus energy helps to serve the morning peak load demand between 7:00 and 10:00 a.m. However, as illustrated in Figure 4, between 8:00 a.m. and 1:00 pm, the diesel generator does not run smoothly. During this period, there are too many start-stop cycles. The diesel generator starts running and goes up to about 50% of its rated capacity and drops to zero again two to 4 h later. From a technical point of view, this causes a loss of efficiency, and from an economic point of view, operation and maintenance costs are higher than expected. In order to improve the model, in further studies, one additional restriction could be introduced, which creates limits for the diesel generator's power generation fluctuations.

Within the limits of the assumptions discussed above, the energy system performs well:

- It follows the demand (Equation 5) and capacity (Equations 2, 3, 4) restrictions.
- The battery is charged when the PV system or the wind turbine produces more energy than demanded.
- When the opposite is true, the battery is discharged, and the diesel generator's power generation follows the remaining load demand.

Case study II: Titumate, Colombia

Titumate is a small village located in the Colombian region of Chocó and in the municipality of Unguía. It is located at the Caribbean coast, near the border to Panama and has about 600 inhabitants [18]. In this village, a solar-diesel hybrid project was recently implemented. This project was of particular interest to us as it enabled the comparison between our model results and a realised system. Thus, it has been selected for the second case study. Moreover, the availability of data for this location is slightly better than for the first case study, which was based on theoretical data. A measured load curve is available. For the weather conditions, the same methodology described previously is used.

The results of the optimisation for Titumate are shown in Figures 3, 8 and 9. First of all, the different characteristic of the load curve attracts attention. In comparison to the load curve of the first case study, which had three daily spikes, the load curve of Titumate has only one evening peak. The electricity demand in the day hours is comparably low. This makes the implementation of a battery storage economically more difficult, as it reduces the number of possible daily cycles.

In Figure 3, the optimal capacities of the different technologies are shown in dependence of the price of diesel fuel. As in the first case study, no wind power is included in the optimal solution. This is due to the poor local wind conditions (average wind speed 3.1 m/s). In comparison with the previous case study in Narendra Nagar, the optimal PV share is already larger than zero with a diesel price of €8 ct/kWh. This is due to the better solar insolation in Titumate, which is nearer to the equator than the Indian site[t]. In Figure 3, the slope of the optimal PV share curve after the initial increase is relatively low, affecting the optimal capacity of the PV share very strongly. The corresponding optimal battery capacity is quite low and rises more steeply only in a range of higher fuel prices. This is, once more, a proof of the fact that PV serves as a fuel-saving technology. The (relatively cheap) diesel generator capacity is not reduced; the batteries are used only for small intra-day fluctuations and for storing the daytime PV electricity for the evening hours with high electricity demand for lighting. Only if diesel prices increased dramatically would the diesel generator capacity be reduced. As the increase of the difference of battery capacity and battery use shows, the utilisation (number of cycles) of the battery bank decreases strongly.

The use of the batteries can be seen in detail in Figure 8, which shows the coverage of the load curve by the different technologies on an exemplary day. For this figure, a diesel price of €9 ct/kWh is assumed. In the morning hours before sunrise, the diesel generator covers 100% of the load. After sunrise, the PV system starts to produce electricity, and the output of the diesel generator is reduced. Around noon, the PV production exceeds the load slightly, and the excess energy is stored into the battery. When the load rises again, the diesel generator is used. At peak load, the electricity stored in the battery is used so that the diesel generator output in the hour with highest load equals the output in the hour with the second highest output (typical peak-

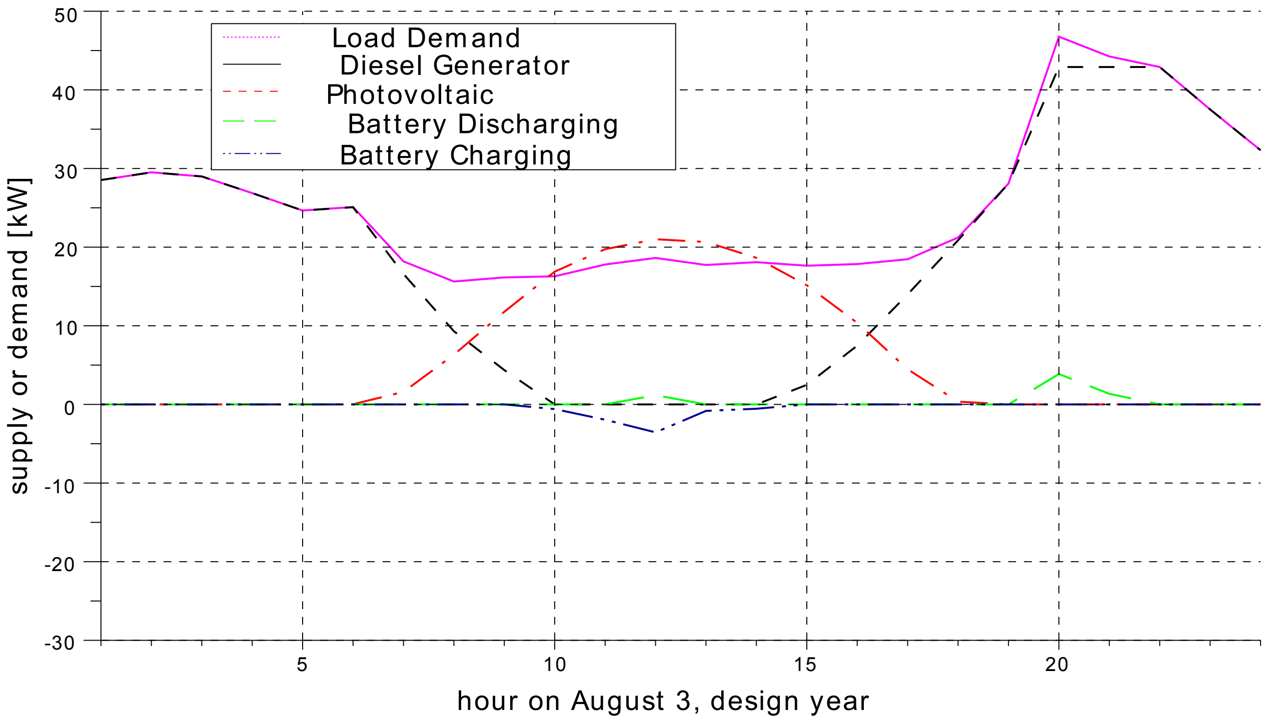

Figure 8 Exemplary optimal power generation distribution strategy for 1 day with solution for an elevated wind speed of 7.5 m/s.

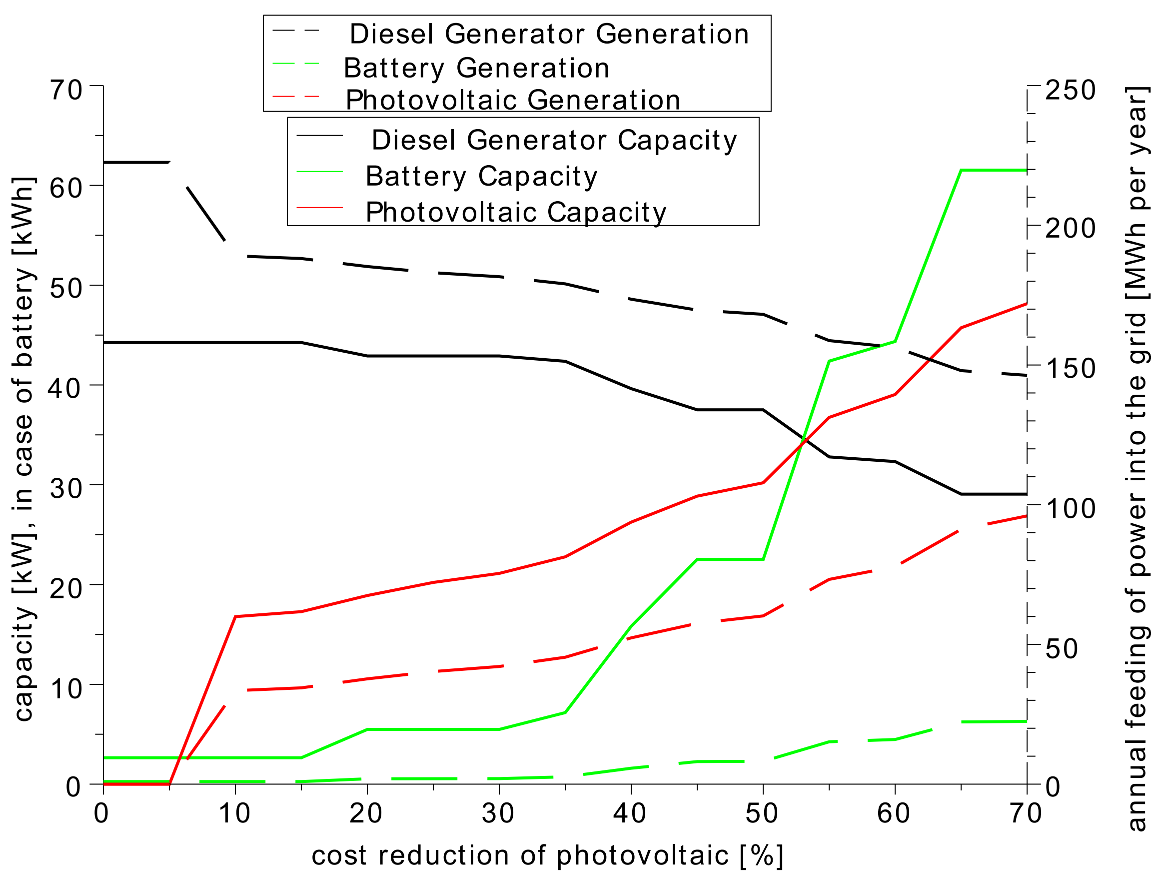

Figure 9 Influence of PV cost reduction on optimal solution.

shaving application; in this way, the capacity of the diesel generator can be reduced slightly).

Figure 9 shows the sensitivity of the optimal solution to the PV installation costs. Because a diesel price of €7.2 ct/kWh is assumed, no share of PV is included at the current PV prices (as was also visible in Figure 3). However, if the PV prices decreased only slightly, the optimal PV generation capacity jumps from 0 to nearly 20 kW. Because of the characteristics of the load curve discussed above, hardly any battery capacity is needed to integrate the PV electricity into the energy system. With PV module costs decreasing further (by about 30%), the optimal battery capacity increases strongly, and the generator capacity can be reduced. Therefore, excess PV electricity at noon can be better utilised and is not wasted.

Comparing the calculated optimal solution with the solution realised in the village of Titumate, the installed PV capacity coincides quite well. At a current local diesel price between €7 and €8 ct/kWh, the economically optimal solution, according to the model, includes a share of PV of around 17 kW_p. In the energy system realised in Titumate, the capacity of the PV array is 9.36 kW_p. With respect to the battery capacity, the realised solution deviates heavily from the model results. The realised battery bank has a capacity of 900 kWh. This surprising solution has been selected not because of economic reasons but because of security of supply and due to political issues. One design condition was that the system is able to operate independently of the generator for a period of 3 days (in order to be able to maintain supply, even if the generator fails or has to undergo revision). A second condition, also politically motivated, restricted the daily operating time of the diesel generator to 4 h.

Conclusions

In the first part of this study, technical possibilities for the satisfaction of remote area electricity demand are assessed. The flexibility and controllability of a three-phase, AC-bus-configured mini-grid solution are the reasons for this design decision. A photovoltaic array, a wind turbine, a diesel generator set and a lead-acid battery bank complete the system design on the power generation side. The decreasing production cost of PV and its usage in solar home systems are discussed. Furthermore, the difference in cost and performance of small-scale wind turbines in comparison to larger turbines is outlined. Additionally, the advantages of controllability and affordability of diesel generator sets are contrasted to their disadvantageous performance at partial load and environmental harmfulness. A general survey considering the possibilities of storing energy technically is followed by a description of the suitability of lead-acid batteries for off-grid use. Hybrid energy systems including more than just one of the mentioned technologies have the advantage of lower cost and a higher security of power supply. The answer to the first research question of finding an advantageous off-grid energy system design is outlined.

The main reason for choosing the method of linear programming is the chance to model and thereby comprehend the basic interdependencies within an energy system. The applied basic economic data are the investment costs in combination with the capital recovery factor, fixed maintenance and operation costs and fuel costs as short-term marginal generation costs. The comparably high investment costs for small-scale wind turbines and the short life span of lead-acid batteries are also noteworthy. From the view of the authors, the necessary linearisation is only problematic in the case of battery lifetime: defining the battery lifetime in terms of years and not in terms of cycles possibly leads to an overestimation of the optimal battery capacity. This is because, in favorable situations, the battery can be operated undergoing more cycles per time interval than in less favorable situations.

In addition to the economic data, the weather data is also exogenous to the model. Hourly meteorological data are hard to obtain for remote areas, and therefore, an estimation method is introduced for the solar radiation and for the wind velocity. In both approaches, hourly weather data of a reference region are linearly charged with validated average site data.

Two case studies are the bases for the analysis of the designed model and for answering the other research questions regarding crucial parameters and basic interdependencies among the technologies:

1. Narendra Nagar in northern India consists of nine villages and has a peak load demand of 113.8 kW. The average wind speed is 5.2 m/s, and 1,800 h/a of PV full-load hours can be achieved. The output of the model's optimisation process is a diesel-PV hybrid energy system including a battery.
2. Titumate in Colombia, close to the Caribbean coast, is a small village, where a PV-diesel hybrid energy system is already installed. Thereby, this case study bears the chance to test the model under measured real load data and to compare the optimal solution with the installed system. The average wind velocity is 3.1 m/s, and up to 2,100 h/a of PV full-load hours can be achieved.

In order to analyse the sensitivity of the model concerning the parameters and to study the cooperation or rivalry of technologies, crucial parameters are varied and their influence on the optimal solution are examined.

Important parameters are:

1. The diesel price represents the most influential parameter for the intensity of usage of solar or wind energy. Keeping steeply increasing fossil fuel prices and decreasing production costs of PV and wind turbines in mind, the suggestion to decision-makers is to open off-grid solutions towards a stepwise integration of renewable energy. Thereby, the more diesel generator electricity can be substituted, the higher the diesel price grows. Particularly, as a fuel-saving technology, PV is economically advantageous.
2. The precise run of the load profiles of the regions is responsible for very different solutions with respect to the suitability of PV and wind energy. A load demand peak at noon increases the competitiveness of PV, while less fluctuations increase the competitiveness of the diesel generator. The consequent advice to system designers is to be as precise as possible in learning the concrete demand situation for an off-grid energy system. There is no general optimal solution; a solution can only be close to optimal with respect to very specific regional and social characteristics of electricity demand.
3. Regarding the influence of investment cost reductions on the optimal solution, the costs of PV and the wind turbine are the crucial parameters. Thereby, the assumed costs of small-scale wind turbines generally are too high for an off-grid integration, while PV definitively has reached off-grid parity in comparison to diesel-only off-grid solutions.
4. For both examined regions, the given weather data were sufficient for the integration of PV, but wind energy was only viable in the Indian case study. A good performance can be expected with an average wind speed of above 6 m/s and more than 1,500 h/a of PV full-load hours.
5. The use of a storage device is currently only economically viable at a very small scale. This is because the costs are still comparably high. Only a technological quantum jump, which decreases the cost dramatically, can make an inter-day storage use economically viable.

The following conclusions describe relevant interdependencies among the technologies derived from the variation of the above mentioned parameters:

1. PV and the wind turbine can interact positively within the energy system. Within the Indian case study, the optimal capacity of PV increases by improving the competitiveness of wind energy[u]. The positive interaction is an effect of a separate usage of the battery by the two technologies.
2. The wind turbine needs more battery capacity than PV in order to close the gap between energy supply and demand.
3. The capacity of the diesel generator is a valuable resource and is substituted only very slowly in comparison to its operation hours. If there is little PV or wind electricity being fed into the grid and if the battery is discharged completely, the diesel generator is able to close the resulting gap. It is replaced by an increasing capacity of the battery only if the diesel price rises substantially and the cost for PV, wind turbines and the battery decrease drastically. Therefore, a total substitution of diesel generators in off-grid energy systems should not be the goal of decision-makers. Hybrid energy systems that include renewable energy help to reduce the cost of electricity much more efficiently.

Endnotes

[a]The necessity of detailed time resolution itself already represents a constraint for the model, complementing the model-intrinsic restrictions discussed later on.

[b]The frequency gives information to the grid manager about the load pattern, so for example, the operation of energy-intensive facilities can be planned in a better way.

[c]These costs are net installation consumer costs in Germany.

[d]A popular example for the use of SHS is rural Bangladesh: SHS are used in off-grid regions above all in order to replace kerosene lamps but also to run TVs. The biggest percentage of SHS was installed by Grameen Shakti, a company of the Grameen Bank: Until March 2011, they had installed an amount of 577,679 SHS in Bangladesh, *cf.* [19]. The heart of the program is the use of micro-credits; Grameen Shakti therefore won among others the *Right Livelihood Award* 2007.

[e]At least for onshore wind turbines, the average installed capacity per wind turbine seems to stabilize at about 2 MW ([20], page 5).

[f]In [10] (page 12), this is the average of the real cost within a field test and the cost given by the manufacturer.

[g]Besides the method of storing energy directly (transforming electricity into a storable form of energy and reconvert it later), it can be done indirectly: avoiding the necessity of storing the energy through demand side management.

[h]Therefore, no power restriction is applied in this study; in detail, high load fluctuations are balanced by a combination of lead-acid batteries with a higher energy density and electrochemical double-layer capacitors with a higher power density.

[i]Their interaction with solar and wind energy is simulated by Younicos AG in Adlershof, Berlin, by means of a sodium-sulfur battery of 1 MW.

[j]The battery's life span is shorter (5 years) than that of the other technologies (20 years) and depends on the number of charging cycles over life and depth of discharge. Nevertheless, a constant life span of 5 years has been applied in order to be able to calculate the capital recovery factor (CRF) for the battery independent of its usage within the system by means of linear programming.

[k]The increase or decrease in population as well as their energy consumption behaviour will change the load profile over time. Consequently, on the long-term view, there is a vast potential for incorrect solutions due to very dynamic social and especially demographical developments. However, this problem is not characteristic for off-grid systems but occurs in every long-term decision.

[l]Clearly, this is a real problem, as in the case of a planned implementation of an off-grid energy system most often there is neither the required data for a remote area nor the time for measuring long-term data.

[m]The capital recovery factor allocates the investment cost annually and is calculated as follows: $\mathrm{CRF} = \frac{i \cdot (1+i)^T}{(1+i)^T - 1}$.

[n]A power output restriction for the battery is not considered to be relevant, as lead-acid batteries are designed for operation with high output. However, if discharged at high outputs, the battery's capacity will diminish.

[o]This is an effect of the assumption of uninterrupted service.

[p]The global horizontal radiation includes direct and diffuse radiation and accounts for a local air mass index as well.

[q]In this way, the battery investment costs appear only once, though the battery is modelled as two technologies.

[r]In the Indian case study, a price increase at the remote area's market of 40% (on the national fuel price) is applied; in the Colombian case study, the site's price is accessible.

[s]According to http://indianpowersector.com/, last retrieved on 28 April 2011, this is the subsidised price for domestic consumers.

[t]The exact assumptions are shown in Table 3.

[u]In a third case study for Conchan, Peru, it was found that the wind turbine and PV can compete as well. If external factors such as the run of the load demand prevent the synergetic effect of the technologies, they compete.

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Authors' contributions

FH and JH conceived the study and drafted the manuscript; FH carried out the modelling. JABG participated in the case study on Titumate in Colombia. GE participated in the study's design and conception. All authors read and approved the final manuscript.

Competing interests

The authors declare that they have no competing interests.

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