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A coupled technological-sociological model for national electrical energy supply systems including sustainability

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Abstract

Global trends in the development and use of electricity utilities and assets are practically irreversible. In industrialized nations, capacity factors have grown so large that users may expect freely available electrical potential energy at all times and in almost all locations. Economically capitalizing on this trend means maximizing energy provision and use to boost gross domestic product growth rates. Electricity is now a basic indicator of social development; it is to the cultural-technological dimension what breathing air is to the physiological-biological dimension, the implication being that sustainable development of provision systems has become a matter of international concern.

This article presents a decision basis for the design of sustainable national electrical energy supply systems, incorporating country-specific boundary conditions in the form of user requirements to be specified by users. The basis is a solution space of technologically possible systems, obtained by combining generalized user requirements and physical limitations to generate the solution states. As all technological options for the system are brought under consideration, this approach represents a comprehensive comparative analysis. The decision process ensues by assigning to each solution state a set of (newly defined) system risk factors. Particular consideration is given to evaluating the system's ability to meet the user requirements, i.e., interruption-free provision. The central benchmark is the technological-economic availability. From this is obtained a sustainability boundary, the boundary between quantifiable and unquantifiable economic loss potentials.

This article deliberately avoids referencing specific technological solutions, with the justification that the basis of the user's decision should be independent of technological considerations. The sole exception is a reference to the currently used technology, which forms the starting point.

Keywords: Utility, Utilitarianism, Electrical energy supply systems, Cellular energy structures, Electricity supply risks, Availability, Sustainability

Background

Existing systems of national electrical energy supply use essentially similar technologies. Each system can be decomposed into the structure of the power stations and of the accompanying grid network.

Though the current technology has unquestionably contributed to economic prosperity, it carries a dominant, unquantifiable systemic risk, i.e., of blackouts. Physically speaking, however, for a general system, the risk of blackouts is not inherent and may be avoided. The principle motivation of this article is to incorporate this unquantifiable risk into a new strategy for comparing possible designs for future systems in terms of their sustainability.

Electrification as a quantifiable social benefit

Electrical energy in a form available to humans does not occur significantly in nature and must therefore be provided artificially. By the end of the nineteenth century, humans had amassed sufficient scientific knowledge to develop the national electrical energy supply systems (EESS). Very fundamental technological innovations were needed to implement a functional system.

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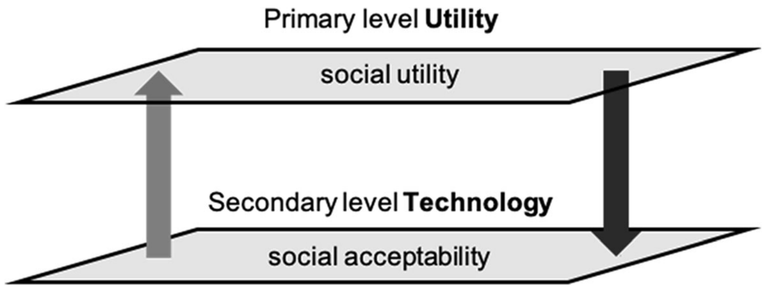

Fig. 1 Model of a social approach to large technological systems

Electrical energy supply

The implementation and sustainable operation of large-scale technological systems depends on majority acceptance by the society in which they are intended to operate. Below, a decision model is developed according to [1], initially consisting of two independent but interacting levels (Fig. 1).

One level decides whether the system has social utility: whether this particular means of electrical energy provision has a positive effect on human well-being, utilitarianism here being the goal-oriented ethics of choice [2].

On the other level, social acceptance is considered, the criteria being investment costs incurred for technology, consequences of technological risk, and consumption of ecological reserves [3].

The levels in the model are not equal. For any system, utility needs be sufficient for consideration, whereas acceptance of the technology is necessary (example: nuclear energy in Germany).

First to be considered is the primary level utility. As early as 1920, the notion of an EESS was semantically raised to an instrument of government: "Communism - that is Soviet power plus electrification of the whole country" [4].

The German electricity industry's development in the years from 1890 to 1950 has been examined by [5]. Across the multi-decadal analysis, they arrive at similar results for different forms of government with their respective political forces and currents. The first working hypothesis is as follows:

- The use of electrical energy is independent of the form of government
- The utility of electrification is economically significant

Economic importance

To test the above working hypothesis about energy consumption, a country's gross domestic product (GDP) is an international economic benchmark that depends on the national consumption of electrical energy (Fig. 2) [6].

The calculated regression function (RF I) shows that rising GDP per capita is characterized by superexponential increase in electricity use. The smallest GDPs per capita in the sample (no use of electrical energy) lie in the region of 1000 USD per capita, and the largest GDPs (for which energy use is unbound) are around 100,000 USD per capita. As energy use rises, its

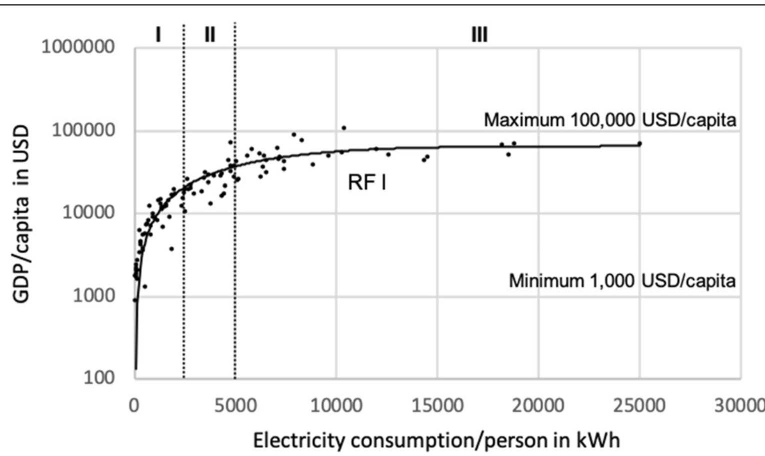

Fig. 2 GDP and electricity consumption in standardized units for a sample of around 100 countries (2015 data). (Zone I 0–2500 kWh. Example countries: Algeria, India, Jordan, Peru, Uzbekistan. Zone II 2500–5000 kWh. Example countries: Brazil, Croatia, Latvia, Poland, Turkey. Zone III over 5000 kWh. Example countries: Germany, France, Kuwait, Norway)

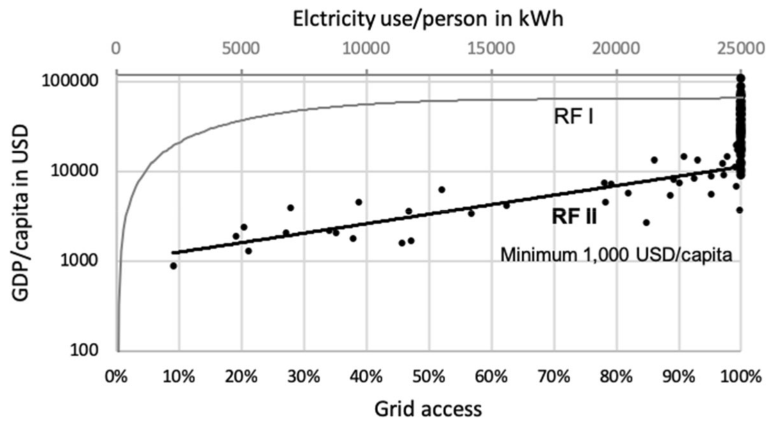

Fig. 3 Normalized GDP and relative electricity grid access for around 100 countries. Grid access data from 2014. Same country sample as Fig. 2

direct influence on GDP decreases sharply. Even in the saturation zone (zone III), though, there is exponential increase.

Figure 2 also shows that as energy use increases, the spread of individual countries' GDP per capita increases significantly, which is reasonably assumed to originate from country-specific factors independent of electrical energy use.

Comparison of the countries with high and low GDP per capita indicate that electrical energy use has a social utility in the utilitarian sense, which, on considering the dataset as a whole, appears to be independent of social structure.

We therefore accept the first working hypothesis.

Key point 1: Provision of electrical energy is associated with utility to national economies that is independent of the type of society

Key point 2: The GDP per capita that is attainable without the use of electrical energy is about 1% of that estimated with unrestricted use of energy and national electrification is therefore economically significant

Current electrical energy supply

In 2013, the volume of electrical energy used worldwide was approximately 20,000 billion kWh, with an average growth of 400 billion kWh per year since 1980 [7]. To provide these amounts of energy, a single technology is currently being used: a combination of centralized large-scale electricity generation plants and comprehensively connected large-scale networks.

Economic importance

Access of a country's population to an electricity grid and the associated opportunities has direct impact on GDP. Figure 3 depicts the situation as described by [6, 8].

The new regression function (RF II) is a standard exponential function over the defined range for relative electricity grid access and GDP. As in RF I, the minimum point is 0% access and USD 1000 GDP per capita. RF I increases most in zone I (0–2500 kWh), and RF II reaches 100% access before the GDP in RF I flattens out. From this, it can be concluded that worldwide use of electrical energy takes place via access to electricity grids.

This is supported by considering a sample from the group of countries with 100% network access.[1] Of global energy consumption, their share alone (2013) is more than 80% [6, 9].

Technology

These energy supply systems are based on Faraday's law of induction; specifically, they are three-phase systems. Existing systems have no significant storage of electrical energy; therefore, the demand must be generated "on-time." The technological basis of this is power-frequency control, with a common operating frequency (e.g., 50 Hz) as the central control variable.

The fundaments are described in [10, 11].

The task of electricity grids is to connect all users with all producers and to transmit the required energy with as little loss as possible. Technologically, this is a major challenge.

Three-phase technology allows the implementation of a fine grid structure that is differentiated by voltage level. The national grid is at the highest voltage. It is a functional link to the lower-level networks and is a

[1] AR/AU/BE/CA/CH/CL/CN/CN/DK/DE/ES/FI/FR/GB/GR/HU/IE/IT/KR/LU/MY/NZ/NL/NO/PL/PT/ RO/RU/STR/ UA/US

[2] Union for the Coordination of the Transmission of Electricity, currently part of ENTSO-E

[3] There are different interpretations of supply quality; from User Requirements for a Cellular Grid, availability is used in the sense of [34]

[4] Unplanned interruptions including all events; the longest was 11.6 min/908 min in 2014

[5] Example: UCTE network area/cf. technological possibility solution space, substantial systemic risk, defining social sustainability

[6] Example: German 'Energiewende' towards wind energy and photovoltaic systems

significant "electrical network node" in the system. In Europe, the national high-voltage grids have been merged to form an international interconnection grid. This integration has lead towards a *European copper plate*, with the largely political goals of increasing the physical exchange of electricity and improving technological supply reliability. One example is the UCTE grid area,[2] which consists of the coupled three-phase networks of 24 countries in central Europe [12]. In this grid region, 440 million users are supplied with electricity: an economic power of 13,000 billion USD (annual figures for 2016). The initiator is the European Union, having outlined the creation of an internal electricity market [13].

Supply reliability and quality[3] is an important system descriptor, for which multiple technical indexes have been devised.

1. The *SAIDI* (System Average Interruption Duration Index) belongs to a group of internationally recognized indicators and describes "...the average interruption in supply per connected final consumer within a calendar year..." [14]. This is a regulatory determination of the supply situation based on past experience. In a 2014 international benchmarking, the SAIDI values were calculated for 27 European countries [15]. The average total annual interruption was 170 min for the 8760 h of a normal year[4]; the average power availability for the end customer is 99.97%. This result can be interpreted favorably, but it raises several questions.
   - Have technological choices led to excessive costs for the users?
   - Are there cheaper technologies with equivalent availability?
   - Is it appropriate to base indicators on past performance?
2. Direct electrical parameters such as short-circuiting [16] also have an effect on the supply quality. The aim is to create a system that is fair for all users, with the highest possible performance. This is a mounting challenge, however, with a growing grid[5] and a change in generation technology, stemming from the move to inverter-based sources and away from direct feed-in via rotating masses [17].[6]
3. Blackouts have the most immediate effect on availability [18]. These occur when fluctuations in the power-frequency control exceed or fall below specified frequency values. In the UCTE grid area, these limits are 50 Hz ± 2.5 Hz [19]. Outside the range, no power plants remain on-grid and the national EESS is functionless. Blackouts can have a range of causes—extensive, sustained blackouts are caused by software and/or hardware irregularities—and risk is inherent to the system, as explored in later sections. Some countries, such as Switzerland, treat blackouts as national hazards [20], paralleling the earlier quotation from Lenin.

Opportunity-risk profile

Grid technology successfully provides electrical energy to users, not only a single national economy but also the world over, and has led to significant global economic growth.

Despite this, the inherent risks can be seen reflected in the supply quality of any one of these grids. Large-scale, sustained functional losses are possible at any time and can have considerable economic impact: "As an Austrian study has found, for an Austria-wide power failure of 24 hours, damage of at least 1 billion Euros, likely several billion Euros" [21].

The opportunity-risk profile of the technology currently used worldwide thus diverges wildly. The economically quantifiable positive effect on national GDP is contrasted with non-quantifiable risk.

Key point 3: Large-scale production plants in combination with large-scale grids are the central technologies of EESS worldwide and are drivers of positive economic development

Key point 4: Inherent systemic risks can at any time result in large-scale failures of unlimited duration

User requirements for a cellular grid

The starting point for the analysis is demand and use. First, EESS user requirements are formulated qualitatively. The requirements are then described quantitatively in cellular structures.

Requirements

Seven ad hoc system requirements are formulated that give structure to the "utility" level of the social acceptance model:

1. A utilitarian approach is taken, due to the large number of users;
2. Electrical energy consumption is meant in the anthropogenic sense;
3. The location of energy use is freely selectable by each user;
4. The time profile of energy use is freely selectable by each user;
5. Each user is limited to a freely selectable, fixed maximum energy use;
6. At any time, energy use equals energy demand;
7. Supply costs are economically minimized.

Later, an ancillary requirement will be derived.

Energy functions

The energy demand function E^D with i, $i_0 \in I$, $i \leq i_0$; $t \in T$, and $\boldsymbol{x}_i \in N \subset R^3$ is defined as

$$\begin{aligned} E^D &: (T \; x \; N) \rightarrow IR^+ \\ &(t, \boldsymbol{x}_i) \rightarrow E^D(t, \boldsymbol{x}_i). \end{aligned} \tag{1}$$

Location vectors $\boldsymbol{x}_i$ uniquely define the ith user location and the time component of individual user behavior. The function can be written as

$$E^D(t, \boldsymbol{x}_i) = E^D_{\max}(\boldsymbol{x}_i) \cdot f^D_i(t), \tag{2}$$

(short form)

$$E^D_i(t) = E^D_{\max,i} \cdot f^D_i(t).$$

with $E^D_{\max,i}$, the maximum energy and $f^D_i : T \rightarrow [0;1]$ a differentiable time function.

The function applies to the primary "utility" level.

There exists no natural energy source with the system requirements, meaning energy must be generated and provisioned anthropogenically. For this, there is the energy supply function E^S:

$$\begin{aligned} E^S &: (T \; x \; N) \rightarrow IR^+, \\ &(t, \boldsymbol{x}_i) \rightarrow E^S(t, \boldsymbol{x}_i), \end{aligned} \tag{3}$$

(short form)

$$E^S_i(t).$$

The variables here have the same meanings as in Eqs. 1 and 2, and the function should be analytic for each location in the time variable.

The supply energy function is part of the secondary "technology" level. The short form will be used in the technological function in Technological Possibility Solution Space.

Microcell

The functions in Eqs. 1 and 3 operate on different levels of the model. Equating them mathematically, we can define the initial balance between them. For each location,

$$E^D_i(t) = E^S_i(t) \rightarrow \int_t E^D_i(t) dt = \int_t E^S_i(t) dt. \tag{4}$$

This defines an energy microcell (*microcell* for short), the smallest energy unit in a national EESS. It incorporates user requirements 2, 3, 4, 5, and 6.

[7]In principle, bundling applies to all aspects from physical connection to the creation of the virtual network. Here, only physical connections are considered (referred to as technological macrocells).

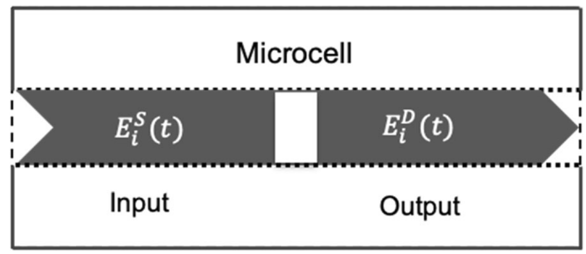

Fig. 4 Elementary energy cell at location $\boldsymbol{x}_i$

Macrocell

To get a handle on the more complex system states, it is greatly helpful to bundle the microcells.[7] Total supply and demand energies for an ensemble can be calculated for a system with j_0 microcells using Eq. 4:

$$E^D_{j_0}(t) = \sum\nolimits_{j=1}^{j_0} E^D_j(t) \text{ and } E^D_{j_0}(t) = \sum\nolimits_{j=1}^{j_0} E^S_j(t). \tag{5}$$

The energy balance equation follows

$$E^D_{j_0}(t) = E^S_{j_0}(t) \rightarrow \int_t E^D_{j_0}(t) dt = \int_t E^S_{j_0}(t) dt. \tag{6}$$

This defines the energy-economic macrocell (or *macrocell* for brevity).

National macrocell

If the ensemble is extended to all i_0 users[8] of a national EESS, a national energy macrocell (or *national macrocell*) is created. This has the following energy relation.

$$\begin{aligned} E^D_{\text{Nat.}}(t) &= \sum\nolimits_{i'=1}^{i'_0} E^D_{i'}(t) + \sum\nolimits_{j=1}^{j_0} E^D_{j_0}(t) \\ &= \sum\nolimits_{i'=1}^{i'_0} E^S_{i'}(t) + \sum\nolimits_{j=1}^{j_0} E^S_{j_0}(t) \\ &= E^S_{\text{Nat.}}(t), \end{aligned} \tag{7}$$

(short form)

$$E^D_{\text{Nat.}}(t) = E^S_{\text{Nat.}}(t) \rightarrow \int_t E^D_{\text{Nat.}}(t)\, dt = \int_t E^S_{\text{Nat.}}(t)\, dt.$$

The energy demand term in the microcells (the output of Fig. 4) is invariant for different system designs. Freedom in the design of the system is captured in the supply term.

Key point 5: Formulation of qualitative user system requirements

Key point 6: Definition and properties of supply and demand energy functions

Key point 7: Definition and properties of energy micro- and macrocells and the national macrocell

[8]i'_0 unbundled microcells, j'_0 bundled microcells, $i_0 = j'_0 + j_0$

Physical limitations and possibilities

The primary "utility" level in the social acceptance model is now somewhat structured and ready to be coupled to the secondary "technology" level. A mezzanine level is introduced to define the coupling (Fig. 5). It should highlight the scientific possibilities and limits that affect both user requirements and possible technological solutions. Bilateral feedback is necessary between primary and mezzanine levels (cf. Electrification as a Quantifiable Social Benefit, the primary level takes precedence). One-way coupling suffices between mezzanine and secondary levels.

The analysis proceeds with aspects from classical field theory, specifically from classical electrodynamics.

The limits of physical possibility for a national electrical energy supply system are determined by the principle of relativity, Maxwell's equations, the associated conservation laws, and macroscopic electromagnetism and the propagation of electromagnetic waves. Detailed treatments of these topics can be found in [22–24]. Here, relevant principles are established such as are necessary here.

The principle of relativity

The most important consequence of relativity for the present case is the finite propagation speed of forces and information (the speed of light in vacuum). It follows that all dynamic physical systems have time lags so that the balance requirement in Eq. 4 yields the relation.

$$E^D(t,\boldsymbol{x}_i)\cong E^S(t',\boldsymbol{x}_i) \tag{8}$$

with

$$t' = t + \Delta t.$$

This formulation also expresses the weighting of the primary and secondary levels: the time shift effect is associated to the supply energy.[9]

Conservation laws

Energy and momentum are conserved in isolated systems, in this case a system of charged particles and electromagnetic fields. Poynting's theorem [25] is a statement of energy conservation and is given here in the form of a balance equation

$$\frac{\partial u}{\partial t} + \nabla\cdot\boldsymbol{S} = -\boldsymbol{J}\cdot\boldsymbol{E} \tag{9}$$

This equation also defines the Poynting vector, which describes the energy flux density of the electromagnetic field.

[9]Ultimate goal of the European copper plate is to allow providers to meet e.g. Warsaw's demand with supply generated elsewhere e.g. in Madrid (UCTE grid area cities). The signal crosses the intervening 4600 km in approx. 15 ms (75% of the wave period at 50 Hz).

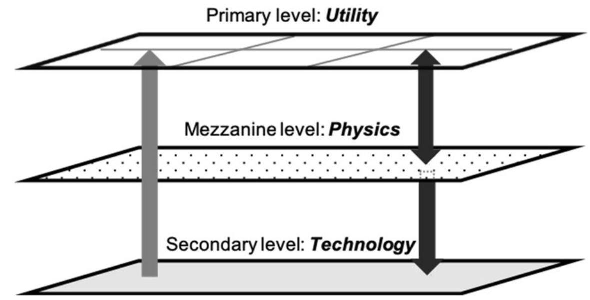

Fig. 5 Extension of the model by structuring the coupling between primary and secondary levels with a mezzanine level

$$\mathbf{S} = \mathbf{E}\times\mathbf{H} \tag{10}$$

In the case of anthropogenic energy transmission by means of electromagnetic waves, radiation losses are of secondary importance due to the low field frequencies. The signal velocity itself is already maximized: it is the speed of light [26]. Conservation of electric charge is encoded in the continuity equation [27]

$$\frac{\partial \varrho}{\partial t} + \nabla\cdot\mathbf{J} = 0 \tag{11}$$

Spatial distances

Spatial remoteness in the sources and sinks of electric charge is not unusual in electrodynamics; indeed, the function of electromagnetic fields is to connect them. The intervening medium determines the speed of light c and, therefore, the signal velocity. The associated time shift Δt is directly proportional to the separation $\Delta\boldsymbol{x}$, with

$$\Delta t = \frac{|\Delta\boldsymbol{x}|}{c}. \tag{12}$$

This inevitable delay contradicts the requirement on the microcells in Eq. 4 that supply and demand energies should have precisely zero time shift.

Parallel circuits

Electricity sources are here assumed to be current sources. Ideal current sources feed current into a connected network independently of the load, or equivalently, can continuously draw from an infinitely large energy reserve without disruption. In reality, ideal sources must be forgone for real sources, with time shifts and finite energy reservoirs [28].

As currents must be superimposed without violating charge conservation (Eq. 11), two-terminal parallel networks must be linear. When current sources (ideal or real) are connected in parallel, their currents add up to a new total current (Kirchhoff's node law), tantamount to a new equivalent current source.

Key point 8: Finiteness of signal propagation in physical systems

Key point 9: Compliance with charge, energy, and momentum conservation laws

Key point 10: Option 1 is to allow spatial remoteness of source and sink; option 2 is to allow parallel connection of sources

Technological Possibility Solution Space

So far, in the social acceptance model, some structure has been given to the primary level, a mezzanine level has been introduced and structured, and the resulting interactions developed. In this section, the secondary level is given some features, resulting in a set of technologically possible solutions.

Supply energy

The structure of the secondary "technology" level is determined by the supply energy E^S. It is analytical in the variable t and may therefore be expanded in a Taylor series (cf. User Requirements for a Cellular Grid, Eq. 3). For each location of x_i,

$$E_i^S\left(t_i'\right) = E_i^{ST}(t) + \frac{\partial E_i^{ST}(t)}{\partial t} \cdot \Delta t_i + \frac{1}{2}\,\frac{\partial^2 E_i^{ST}(t)}{\partial t^2} \cdot \Delta t_i^2 + R_3\left(t_i'\right). \tag{13}$$

Definitions:

- Time shift $\Delta t_i = t_i' - t$ in the ith microcell;
- The zeroth order term describes a constant supply energy level at time t;
- The first order term contains the temporal derivative of energy, i.e., power $P^S(t, x_i)$;
- The second order term contains the power dynamics $\dot{P}_i^S(t)$; and
- The third term contains higher order derivatives.

A national EESS is described by the following system of equations, in which higher order terms are neglected

$$\begin{array}{llll} E_1^D(t) \cong E_1^S\left(t_1'\right) \cong & E_1^{ST}(t) & + \dfrac{\partial E_1^{ST}(t)}{\partial t} . \Delta t_1 & + \dfrac{1}{2}\dfrac{\partial^2 E_1^{ST}(T)}{\partial t^2} . \Delta t_1^2 \\ \vdots & \vdots & \vdots & \\ E_{i_0}^D(t) \cong E_{i_0}^S\left(t_{i_0}'\right) \cong & E_{i_0}^{ST}(t) & + \dfrac{\partial E_{i_0}^{ST}(t)}{\partial t} . \Delta t_{i_0} & + \dfrac{1}{2}\dfrac{\partial^2 E_{i_0}^{ST}(t)}{\partial t^2} . \Delta t_{i_0}^2 \end{array} \tag{14}$$

Each equation describes a microcell. Macrocells can then be created by combining corresponding rows. The right hand side of the system of equations encodes the possible technological design variables (cf. User Requirements for a Cellular Grid). For Germany, i_0 is approximately 45 million.

The structure variable

To ensure satisfaction of Equation System 14, five system-defining technological structural variables (S1–S5) are devised.

S_1–time shift, $\Delta t \to 0$.

The lag between energy use and supply has two components: relativistic, Δt_r, and non-ideal current source effects, Δt_S.

For the ith microcell, linear superposition Δt_i gives

$$\Delta t_i = \Delta t_{r_i} + \Delta t_{s_i}. \tag{15}$$

A technological macrocell is a collection of j_0 microcells and has a total time shift Δt_{j_0}, which may be compared with the equivalent value for the microcells $\sum_{i=1}^{j_0} t_i$. The smaller of the two better fulfills the user requirements.

S_2–stationary system states, $E_i^{ST}(t) = c_i$

Any energetically possible stationary state of a microcell can be demanded at a given time. Equation System 14 has the following condition for stationary states

$$E_i^D = E_i^S = E_i^{ST} = c_i. \tag{16}$$

Kirchhoff's node rule implies that superposition applies to macrocells, written as

$$E_{j_0}^{ST} = \sum\nolimits_{j=1}^{j_0} E_j^{ST} = \sum\nolimits_{j=1}^{j_0} c_j. \tag{17}$$

S_3–power output of current sources, $P_i^S(t) \to \infty$

The finite power output of real sources varies over finite time intervals as the temporal gradient of the supply energy. The greater the gradient at a stationary operating point $E_i^{ST}(t)$, the shorter the necessary adjustment time interval Δt_{s_i}. The source output must be technologically forced towards the ideal value. This applies to microcells and macrocells and is therefore relevant to the whole system (cf. Current Electrical Energy Supply, reducing the system's short-circuit power).

S_4– power dynamics of current sources, $\dot{P}_i^S(t) \to \infty$

The requirements on the power dynamics of the sources mirror those from S_3. Deviations are small, bounded by the time adjustment interval Δt_i^2. To achieve significant dynamics is a particular technological challenge under the formulated economic boundary conditions. The situation applies equally to microcells and macrocells and is, like power output, a key element in system choice.

S_5–maximum power of a national EESS, $\wp_{\text{Nat.}}^D$

Making use of the mean value theorem, the energy demand function defined in Eq. 2 must have a maximum. For the ith microcell, with $t_0 \in T$,

$$P_i^{D_{\max}} := E^D_{\max,i} \cdot \frac{\partial f_i^D(t_0)}{\partial t}, \tag{18}$$

and Eq. 4 gives the power relation

$$P_i^{D_{\max}} = P_i^{S_{\max}}. \tag{19}$$

Free individual user behavior is given by a function f_i^D, and the system must be capable of providing the

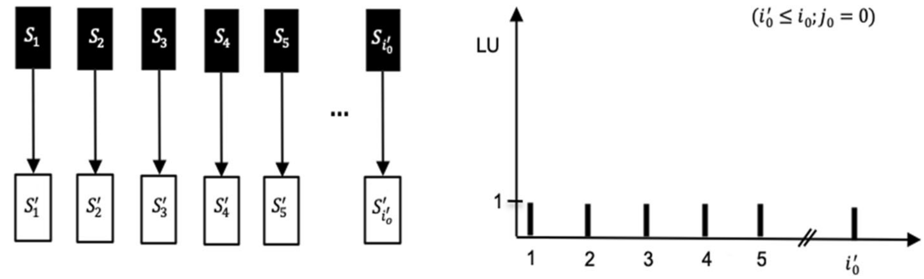

Fig. 6 Base module I: schematic and associated grid spectrum

maximum required power at any time. Because hardware should lie comfortably in the realm of adequacy for any demand placed on it, hardware is the main driver of cost.

For a technological macrocell consisting of j_0 microcells, the balance equation is obtained by summation

$$\wp^D_{\text{macrocell}} = \sum\nolimits_{j=1}^{j_0} P_j^{D_{\max}} = P(j_0) = P(k_0) \\ = \sum\nolimits_{j=1}^{j_0} P_j^{s_{\max}} = \wp^S_{\text{macrocell}}. \tag{20}$$

The two physical possibilities for the system discussed earlier are a parallel connection of sources and spatial remoteness of source and sink. These are two technological degrees of freedom in the macrocell and degrees of system design freedom—together with options for the number and size of sources. If there are k_0 current sources, only the power relation $P(j_0) = P(k_0)$ applies and the freedom lies in k_0, with $1 \leq k_0 \leq j_0$. For a national macrocell,

$$\wp^D_{\text{Nat.}} = \sum\nolimits_{j=1}^{j_0} P_j^{S_{\max}} + \sum\nolimits_{i'=1}^{i_0'} P_{i'}^{D_{\max}}. \tag{21}$$

The above five parameters comprise the initial structure vector S^*

$$\boldsymbol{S}^* = (S_1; S_2; S_3, S_4; S_5) \tag{22}$$

Base modules

Energy is exchanged between source and sink in the form of electromagnetic waves, which require material connection suitable for the high energy fluxes of an EESS [29]. Physically, this can be interpreted as meaning that upon request, a Poynting vector[10] is transmitted along the conductive material to the destination. Such a structure is referred to as a power grid or simply grid. Grids are defined by paths in the three-dimensional Euclidean vector space, mathematically described by a metric space and its special properties (cf. Additional file 1).

A second working hypothesis can now be formulated:

- Operation of the system without a grid and without source bundling is impossible
- Operation of the system with a grid but without source bundling is useless

As a result, the two physical options are combined into a single usable technology, to which electricity generation is primary and the grid is secondary. Seen economically, this is a two-stage production process whose sub-processes are technologically different.

The following section introduces two base modules from which each technological system state can be generated (cf. Additional files 1 and 2).

Base module I

The base module I consists only of singular microcells i_0' so that $i_0' \leq i_0; j_0 = 0$. Source requirements are given by Eqs. 4, 8, and 19. For the ith microcell, the source is at $\boldsymbol{x}'_{i'}$ and the sink at $\boldsymbol{x}_{i'}$ with $\boldsymbol{x}'_{i'} \approx \boldsymbol{x}_{i'}$. The network is a microgrid within the metric space $(N_{i'}, d_{i',|.|})$, with an associated conductivity function.

The mathematical concept of connectedness of subsets underlies the grid structure. For base module I, each individual microcell is connected, and the i_0'-microcell ensemble is pairwise disconnected.

Grid spectra show connection lengths within a grid. Figure 6 shows the system structure of base module I and the resulting grid spectrum.

Base module II

Base module II consists of bundled microcells j_0 with $i_0' = 0$; $j_0 \leq i_0; k_0 < j_0$. Meeting the source requirements proceeds differently for base module II than for base module I. The 1:1 source-sink fraction in base module I is replaced with k_0, $k_0 < j_0$, new equivalent current sources based on parallel connections. The equivalent current sources are located at $\boldsymbol{x}_1, \ldots, \boldsymbol{x}_{k_0}$ and the sinks at $\boldsymbol{x}_1, \ldots, \boldsymbol{x}_{j_0}$. Location vectors are unique, and the distances are, as before, significant. The network is a macrogrid. The grid is carried by the metric space (N_{n_0}, d_{n_0})

[10] cf. Physical Limitations and Possibilities; Appendix IV. Here only the magnitude of the Poynting vector was used.

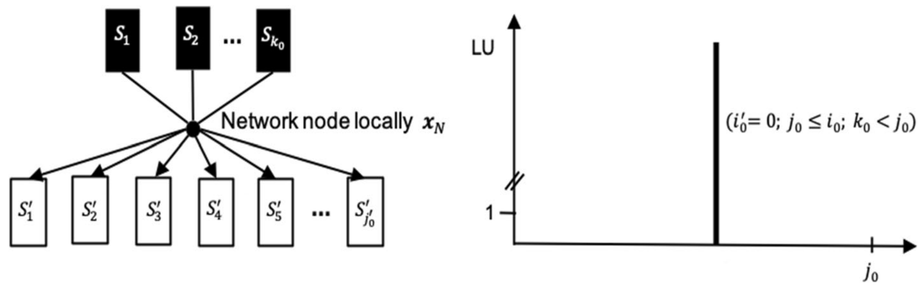

Fig. 7 Base module II: schematic and associated grid spectrum

based on the French railway metric, with additional location vector $\mathbf{x}_N$. In this metric, the ensemble containing j_0 microcells and k_0 current sources is globally connected. Figure 7 depicts a schematic of base module II and the associated grid spectrum.

For this module, some additional electrical observations may be made. First, note that the total supply energy is obtained from a single equivalent current source $k_0 = 1$. Thus, there exists at least one network node. At all times, the entire energy flux of the macrocell is passing through this node. The grid structure corresponds to one such equivalent node (cf. Current Electrical Energy Supply).

System degrees of freedom: energy demand

The user requirements and technological degrees of freedom add their own dimensions to system design. The choice of any technological "option" has associated sociological consequences, as can be demonstrated with the base modules.

Base module I

All microcells are by design electrically independent of each other, with the sociological consequence that the decision about a microcell's technological design lies exclusively with the microcell user. He is thus solely responsible for the business costs of his decision.

There is no technological-economic socialization.

Base module II

In this module, all microcells are connected to form a macrocell, meaning all microcells are electrically interdependent. The sociological effect of this is to pass decision-making authority from the individual user to a third party. This determines the business characteristics, and resulting costs remain in the user group.

In this module, there is technological-economic socialization.

The above are structural features of the secondary "technology" level.

System states and technological solution space

System states describing technologically possible configurations for a national system are denoted by state vectors (cf. Additional file 2). The state vector components are, for now, the number of microcells not connected in parallel i'_0, the number of technological macrocells n_0, the number of parallel-connected microcells j_0^*, and the number of parallel-connected equivalent current sources k_0^*. They are real vectors in the set.

$$\Omega_{\boldsymbol{u}} := \left\{ \boldsymbol{u} \in R^4 \mid \boldsymbol{u} = \left(i'_0; n_0; j_0^*; k_0^* \right) \right\}. \tag{23}$$

The state space Ω_u is the solution space of all technologically possible configurations. Two of the state variables, the numbers of parallel-connected microcells and of equivalent current sources, are functionally dependent, so for constant j_0^* and k_0^*, cellular variety is possible across the n_0 technological macrocells.

This manifests as additional state vectors, so-called fine structure vectors. Properties of Ω_u can be deduced. Like the base modules, each state vector has a grid spectrum.

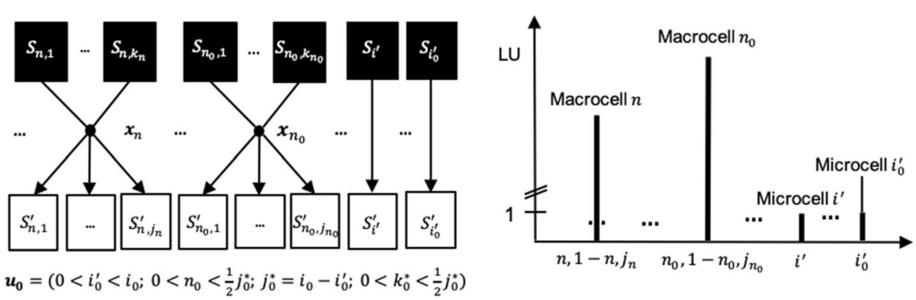

Fig. 8 National EESS (u_0)—schematic and associated grid spectrum

Figure 8 shows a polychromatic-state based on the combinations of the base modules. The possible states $\boldsymbol{u}_0$ are members of Ω_{u_0}.

In addition, there are two further monochromatic states given by the base modules I and II as national EESS (Table 1).

The technological solution space is built from these subsets

$$\Omega_u = \Omega_{u_0} \cup \Omega_{u_I} \cup \Omega_{u_{II}} \tag{24}$$

and the component representation of a general vector $\boldsymbol{u} \in \Omega_u$ is

$$\boldsymbol{u} = \left(0 \le i_0' < i_0; 0 \le n_0 < \frac{1}{2} j_0^*; j_0^* = i_0 - i_0'; 0 \le k_0^* < \frac{1}{2} j_0^*\right). \tag{25}$$

A sensible question at this stage is whether Ω_u is mathematically complete, that is, whether all possible states are in Ω_u. Without going into a rigorous mathematical proof, completeness will be demonstrated by means of the grid spectra and the state vectors.

The grid spectra of the base modules shown in Figs. 6 and 7 represent the extreme states. The spectra do not fundamentally change in transition to monochromatic states, implying the monochromatic states are extreme. Since u_0 is any polychromatic state, the associated spectra must lie between these extremes.

The state vector components are indexed with natural numbers; since all indexes are defined by their being possible, the resulting state vector set is also complete.

Therefore the technological solution space is complete, providing the basis for a decision on the preferred national EESS now looking to the user requirements.[11]

Key point 11: Declaration of technological structural variables

Key point 12: Introduction of base modules

Key point 13: Definition of a power grid structure and associated grid spectra

Key point 14: Reduction of national EESS to state vectors and their solution set

Substantial systemic risk

Assessing the systemic risk of similar technologies sometimes reveals significant variation. The analysis and evaluation of risks in engineering is therefore extensively researched [30].

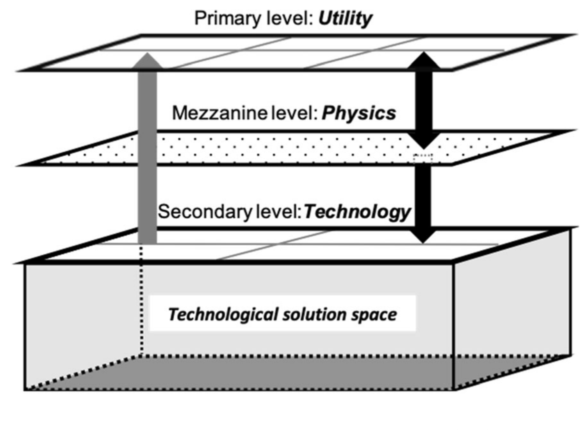

Fig. 9 Model extension by the technological solution set (Ω_u)

The present analysis predicts fundamental systemic effects of risk in national EESS. Central to this assessment is a substantial system risk with two subcategories:

1. Sudden change from normal operating state to a system OFF state
2. Duration of a system OFF state

The substantial systemic risk has an associated likelihood r_s; there are also likelihoods for the two subcategories, r_1, r_2:

$$r_s = r_1 \cdot r_2. \tag{26}$$

Risk factor r_1

In an EESS, the sudden change from a normal operating state to a system OFF state implies rapid loss of function in a connected electrical element; here the failure of a national macrocell is considered. Combining Eq. 7 and the failure factor $\mu \in [0, 1]$, with $t_0, t_0' \in T$, yields

Table 1 Component representation of the state vectors of a national EESS

$\boldsymbol{u}_0 \in \Omega_{u_0}$	$\boldsymbol{u}_0 = (0 < i_0' < i_0; 0 < n_0 < \frac{1}{2} j_0^*; j_0^* = i_0 - i_0'; 0 < k_0^* < \frac{1}{2} j_0^*)$ Basis: polychromatic system consisting of base modules I and II
$\boldsymbol{u}_I \in \Omega_{u_I}$ and $\boldsymbol{u}_I \notin \Omega_{u_0}$	$\boldsymbol{u}_I = (i_0' = i_0; n_0 = 0; j_0^* = 0; k_0^* = 0)$ Basis: monochromatic system from base module I
$\boldsymbol{u}_{II} \in \Omega_{u_{II}}$ and $\boldsymbol{u}_{II} \notin \Omega_{u_0}$	$\boldsymbol{u}_{II} = (i_0' = 0; n_0 = 1; j_0^* = i_0; k_0^* \ge 1)$ Basis: monochromatic system from base module II

[11]The EESS described in Current Electrical Energy Supply has excess production capacity. It is not in u as it does not meet the user requirements. Generation overcapacities are discussed in Substantial Systemic Risk.

$$\frac{\partial E^D_{\text{Nat.}}(t_0)}{\partial t} \rightarrow -\infty, \text{and } E^S_{\text{Nat.}}\left(t_0^{'}\right) < \mu \cdot E^D_{\text{Nat.}}\left(t_0^{'}\right), \quad (27)$$

given that the signal propagates with the speed of light in the medium and where a 5% upper bound is been assumed ($\mu \in [0; 0.05]$). System states where at least 0.1% of the total users (total > 10^6) are modeled independently and are collective cell structures, subject to statistical conditions.

Determining the risk factor

1.1	$\boldsymbol{u}_I \in \Omega_{u_I}$	Basis: monochromatic system
	Risk factor	$r_{1,\,1} = 0$

The likelihood for microcell failure is taken as $p_i = \frac{5}{365}$, an interruption likelihood of 5 days per year. The lower limit for a national system with $i_0 = 10^6$ users is the statistical failure of about 15,000 microcells per day, with a failure rate of 1.5%. This is assumed to represent 1.5% of energy demand. Then, $E^S = 0.985 \cdot E^D$ and, according to Eq. 27, the system is not in the OFF state. The extreme situation would be for all cells to switch to the OFF state at the same time. The likelihood of this is

$$P_{i_0} = p_i^{i_0} = \left(\frac{5}{365}\right)^{10^6} = 0. \quad (28)$$

A national EESS in system state u_I cannot entirely lose functionality in the sense of a system OFF state.

1.2	$\boldsymbol{u}_{II} \in \Omega_{u_{II}}$	Basis: monochromatic system
	Risk factor	$r_{1,\,2} = 1$

The macrocell is completely connected and thus not a statistical collective. Maximum loss of function occurs when the node location vector is absent from the base set (cf. Technological Possibility Solution Space; Additional file 1). Propagation occurs at the speed of light in the relevant medium, as has been observed in real interruptions.[12]

A national EESS in system state u_{II} can enter a system OFF state (Fig. 9).

1.3	$\boldsymbol{u}_0 \in \Omega_{u_0}$	Basis: polychromatic system
	Risk factor	$0 < r_{1,\,3}(u_0) < 1$

The distribution of micro- and macrocells in a system state underlies the overall risk factor. The microcell contribution can be assumed to be zero, as it is guaranteed to be smaller than in the state ($\boldsymbol{u}_I$). The contribution from macrocells is again determined by their number and size (cf. Defining Social Sustainability). The risk factor $r_{1,\,3}$ depends on the system state $\boldsymbol{u}_0$.

To set bounds on the risk factor, two boundary cases are considered:

- In the first case, the number of independent microcells approaches $i_0^{'} \rightarrow i_0$ so that the system approaches the state $\boldsymbol{u}_{\text{I}}$, i.e., $r_{1,\,3} \rightarrow 0$
- In the second case, the macrocells approach $n_0 \rightarrow 1$ and the number of non-parallel microcells disappears, $i_0^{'} \rightarrow 0$, so that the system approaches u_{II}, i.e., $r_{1,\,3} \rightarrow 1$

Risk factor r_2

The next subject is the duration of OFF states, that is, the period of time from the system entering a function-loss OFF state to the recovery of normal operation. Here, OFF states that last longer than 24 h are considered. Such interruptions are caused by fundamental system impairments. Equation 27 results in the following condition

$$\forall t \in [t_0; t_1) : E^S(t) < \mu \cdot E^D(t) \text{ and } t_1 > t_0 + 24h. \quad (29)$$

There are various ways of recovering operation (Fig. 10).

- Redundancies are existing system parts that can compensate for planned or unplanned failures. Redundancy support is generally effective for less than 24 h. Redundancies are not further considered here.
- Parallel systems are entire existing systems that are capable of establishing a regular operating state without accessing the OFF-state system. Recovery is exponential for large technological systems with a time constant τ. Recovery time is greater than 24 h.
- Reparation is the restoration of an initial state and is divided into two model stages. Until t_1, defective facility elements are recovered with no intervening supply (dead time). After t_1, further elements are repaired and supply is exponentially resumed. Recovery time is greater than 24 h.

Dead times and time constants characterize parallel systems and recovery strategies.

Determining the risk factor

2.1	$\boldsymbol{u}_I \in \Omega_{u_I}$	Basis: monochromatic system
	Risk factor	$r_{2,\,1} = 0$

The risk factor $r_{1,\,1}$ of a national macrocell consisting only of microcells is zero, and the system cannot change to a system OFF state.

[12]UCTE grid area interruption of 4.11.2006: severe frequency drop originating in West zone; $\frac{\Delta f}{\Delta t} \approx \frac{1\,Hz}{30\,s}$ [35]. Interpolating from the low-limit frequency of 47.5 Hz indicates that for up to 90 s, all power plants were disconnected.

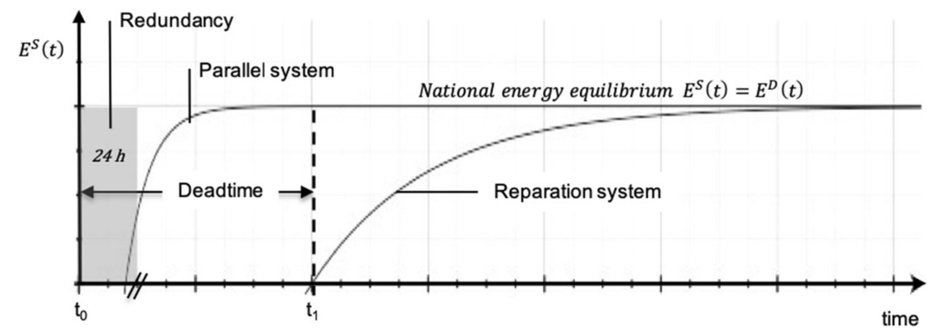

Fig. 10 Recovery paths to the national energy equilibrium level

2.2	$\boldsymbol{u}_{II} \in \Omega_{u_{II}}$	Basis: monochromatic system
	Risk factor	$r_{2,\,2} = 1$

A national technological macrocell may completely lose function and can therefore satisfy Eq. 28. A parallel system in this case would be a second national electricity grid capable of assuming the supply task given the requirements above. The grids operate under conditions of natural monopoly, in which the cost function is subadditive [31]. This means that for economic reasons, there is no parallel option for a national EESS.

The remaining strategy is reparation. For an order-of-magnitude estimation, only the dead time has to be considered. If the fundamental impairments are mechanical in nature, a low estimate for dead time is at least 6 months. This represents manufacturing or production time for the failed plant elements and is already significant[13]; the entire repair takes significantly more time. The conclusion is that a national EESS in u_{II} can assume a system OFF state of inestimable duration.

2.3	$\boldsymbol{u}_0 \in \Omega_{u_0}$	Basis: polychromatic system
	Risk factor	$0 < r_{2,\,3}(u_0) < 1$

A starting point is the properties of $r_{1,\,3}$. Boundary conditions can be deduced as follows:

- In one case, the number of independent technological microcells approaches the number of users $i_0^{/} \to i_0$; states with $r_{2,\,1}$ approach zero
- In another case, the system approaches a national macrocell state $n_0 \to 1$. The system approaches the state $r_{2,\,2}$, i.e., $r_{2,\,3}$ tend to 1

Here, too, risk factors depend on system state.

[13]For Republic of Austria, quantifiable damage would amount to at least € 180 billion [21].

Risk factor r_s

Table 2 lists substantial risk for system states $(u_{\mathrm{I}}, u_{\mathrm{II}})$. The state consisting only of a national macrocell has the highest risk—the system which fully utilizes the two physical options (cf. Current Electrical Energy Supply; this is the case for the current EESS). All other states have lower risks, but it is nonetheless a broad spectrum. A national EESS consisting only of individual microcells has zero substantial risk.

As the substantial risk factor distinguishes system states technologically, it represents another structural variable (cf. Technological Possibility Solution Space).

S_6–substantial risk factor, $r_s \to 0$

The new structure vector describes the system technologically, specifically the degree of interconnections in the grid structure in a national macrocell. It emphasizes functional loss of macrocells.

The substantial risk factor $\boldsymbol{S}^*$ is extended by component S_6 to complete the technological structure vector $\boldsymbol{S}^{**}$.

Key point 15: Risk determination for a sudden change from normal operating state to system OFF state

Key point 16: Risk determination for the duration of a system OFF state

Key point 17: Risk determination for an existing substantial system risk

Key point 18: Definition of substantial risk factor as sixth technological structural variable for national EESS

Table 2 Substantial risk factors for system states

System state			
Risk factor	$\boldsymbol{u}_I \in \Omega_{u_I}$	$\boldsymbol{u}_{II} \in \Omega_{u_{II}}$	$\boldsymbol{u}_0 \in \Omega_{u_0}$
r_1	$r_{1,\,1} = 0$	$r_{1,\,2} = 1$	$r_{1,\,3}(u_0) \in (0;1)$
r_2	$r_{2,\,1} = 0$	$r_{2,\,2} = 1$	$r_{2,\,3}(u_0) \in (0;1)$
$r_s = r_1 \cdot r_2$	$r_{s,u_I} = 0$	$r_{s,u_{II}} = 1$	$r_{s,u_0} \in (0;1)$

Defining social sustainability

System states

A sustainability dimension is incorporated through a new technological and economic availability. Availability is understood in the sense of [32] and represents the utilization potential of the system. The sustainability component now added to the state vectors $u \in \Omega_u$ equals the product from the availability parameter ν_{tv} and the total load for a national macrocell $E^D_{\text{Nat.}}(t)$ from Eq. 7. The new sustainability component is

$$\boldsymbol{\nu} = \left(i_0^{'}, n_0; j_0^*; k_0^*; \nu_{tv} \cdot E^D_{\text{Nat.}}(t)\right) \tag{30}$$

with

$\boldsymbol{\nu} \in \Omega$ and $\Omega_u \subset \Omega$.

Availability is related to risk through the substantial risk factor r_s:

$$\nu_{tv} = 1 - r_s \tag{31}$$

with

$\nu_{tv} \in [0; 1]$.

The sustainability of a load on a national EESS now depends on the technology used. The limiting cases (cf. Table 2) of substantial risk ($r_s = 0$; $r_s = 1$) are sustainable for $\boldsymbol{\nu}_{\text{I}} = (i_0^{'}, n_0; j_0^*; k_0^*; 1)$ and unsustainable for $\boldsymbol{\nu}_{\text{II}} = (i_0^{'}, n_0; j_0^*; k_0^*; 0)$.

Existence of a boundary between states with quantifiable risk and non-quantifiable risk is implied by the completeness of the technological solution set Ω_u (Fig. 11).

The determination of this sustainability limit (sustainable availability limit) is an economic problem (cf. Appendix 1).

Due to considerable variation in the number and size of macrocells in the various system states, $\boldsymbol{u}_0 \in \Omega_{u_0}$, there is a spatial dimension to sustainability. The sustainability components of distinct macrocells must be distinguished, on the basis of Eq. 5. This means that

$$\nu_{tv} \cdot E^D_{j_0}(t) = \sum\nolimits_{n=1}^{n_0} \nu_{tv,n} \cdot E^D_n(t), \tag{32}$$

which introduces regional sustainability into the national EESS. Differences can have historical reasons or arise from future-oriented processes (innovation, transformation).

Energy quantities and system states

The energy balances given by the user requirements cannot be ideally satisfied in the operation of real energy cells. Deviations due to supply reductions or interruptions due to faults are quantitatively expressed by a supply factor $\lambda \in [0; 1]$.

For the ith user of the ith microcell, the individual energy function becomes (for simplicity, $t^{'} = t$):

$$E^S_i(t) \geq \lambda^-_i \cdot E^D_i(t) \tag{33}$$

with

$\lambda^-_i \leq 1$.

The economic interests of a national macrocell are expressed by the energy function

$$E^S_{\text{Nat.}}(t) \geq \lambda^-_{\min} \cdot E^D_{\text{Nat.}} \tag{34}$$

with

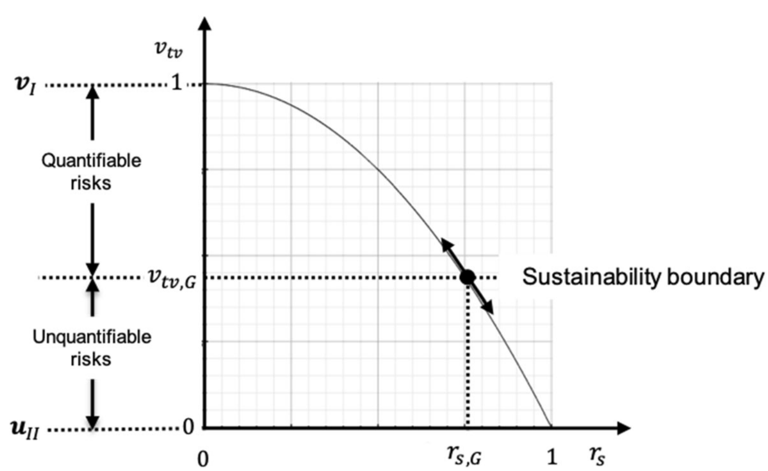

Fig. 11 Boundary due to the sustainability limit in the state model of an EESS. As an example, (r_1, r_2) are assumed linear

$\lambda^-_{\min} \leq 1.$

A utilitarian definition of utility implies a boundary condition

$$\lambda^-_{\min} \leq \lambda^-_i \leq 1. \tag{35}$$

$\lambda^+_i, \lambda^+_{\max} > 1$ are states with generation overcapacities; they do not alter r_s. The energy demand $E^D_{\text{Nat.}}(t)$ from Eq. 33 can be approximated by the product of a freely selectable standardized distribution function $h(t)$ and an annual reference energy quantity $E^D_{T_{\text{Ref}}}$. The energy relation for supply energy is then

$$E^S_{\text{Nat.}} \geq \lambda^-_{\min} \cdot E^D_{\text{Nat.}}(t) \approx \lambda^-_{\min} \cdot h(t) \leq \cdot E^D_{T_{\text{Ref}}} \tag{36}$$

This is the basis for predicting the demand. Information about the supply status in the microcells is necessary for the operation of a national EESS. In addition to passive analysis, active prognoses can be made about future energy demand and microcells can be centrally controlled in an interruption (i.e., a complete loss of function/blackout). For this, there is smart meter technology,[14] now a key EU energy policy issue [33].

The energy quantities from Eq. 33 need to be connected to the sustainable system states of Eq. 29. This is expressed in the coupling relation

$$\lambda^-_{\min} = g(x) \cdot v_{tv}(r_s) = g(x) \cdot (1 - r_s) \tag{37}$$

with weighting function $g(x)$, shown in Fig. 12. For simplicity, $g(x) \equiv 1$.

The above builds a technological-economic foundation for the design of a sustainable national EESS.

The process for defining boundary conditions should have at its center utilitarian benefit and can be devised by the user community. Ensuring the transparency of this process is a social, economic, and technical challenge which needs further investigation (Appendix 2). The prerequisite for sustainability is that system variability must be contained within the technology and not affect the energy demand and supply.

[14]Functions of a "smart meter":1. Determine current demand2. Record current supply3. Predict future demand4. Monitor each user's availability5. Active system control in the grid areas, in particular, for macrocell failure, e.g., blackout control

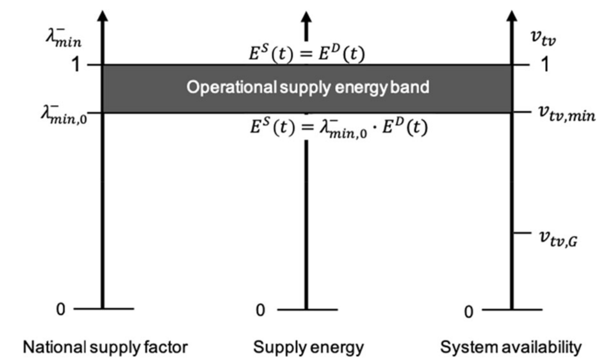

Fig. 12 Energy band of a national macrocell at a given time

Appendix 1

Table 3 List of abbreviations

EESS	Electrical energy supply system
RF	Regression function
$I \subset N_0$	With $i, i_0, i', i'_0 \in I$ and $i, i'_0 \leq i_0, i' \leq i'_0$
$T \subset R$	With t, t', Δt, Δt_R, t_A, t_E, $\tau \in T$ and $\Delta t = t' - t$
$N \subset R^3$	With $x_i, x'_i, x_j, x_{n_0} \in N$, pairwise coprime
$E^D(t, x_i)$	Energy demand function; short form $E^N_i(t)$
$E^D_{\max}(x_i)$	Maximum energy demand; short form $E^N{}_{\max,i}(t)$
$f^D_i(t)$	Time-dependent behavior of user i
$E^S(t, x_i)$	Energy supply function; short form $E^B_i(t)$
$E^D_{j_0}\frac{(t)}{E^S_{j_0}}(t)$	Bundled energy demand/supply of j_0 microcells
$E^D_{\text{Nat.}}\frac{(t)}{E^S_{\text{Nat.}}}(t)$	Energy demand/supply function of a national macrocell
u	Energy density of the electromagnetic field
$\boldsymbol{S}$	Poynting vector
$\boldsymbol{J}$	Current density
$\boldsymbol{E}$	Electric field strength
$\boldsymbol{H}$	Magnetic field strength
ρ	Electrical charge density
$\frac{c_0}{c}$	Speed of light in vacuum/in a medium
E^{ST}_i	Stationary supply component of the ith microcell
$c_i \in R$	Stationary state constant of the ith microcell
$E^{ST}_{j_0}$	Stationary supply constant of a macrocell
$P^S(t, x_i)$	Output power of the ith source; short form $P^B_i(t)$
$P^{D_{\max}}_i$	Maximum load (power consumption) of the ith microcell
$P^{S_{\max}}_i$	Maximum supply (power generation) of the ith microcell
$\dot{P}^S(t, x_i)$	Power dynamics of the ith microcell; short form $\dot{P}^B_i(t)$
$P(j_0)$	Total power from j_0 microcells
$P(k_0)$	Total power from k_0 current sources

Table 3 List of abbreviations *(Continued)*

$\wp^N_{\text{macrocell}}$	Maximum load of a macrocell, analogous to power generation
$\wp^N_{\text{Nat.}}$	Maximum load of a national macro cell, analogous to power generation
$S_1, \ldots, S_5$	Structure variables, components of S^*
$\boldsymbol{S}^* \in R^5$	Technological structure vector
Δt_{r_i}	Time shift in the *i*th microcell due to the relativity principle
Δt_{s_i}	Time shift in the *i*th microcell considering real sources
Δt_{j_0}	Total time delay within a macrocell of j_0 microcells
$J \subset N_0$	With $j, j_0 \in J$ and $j \leq j_0$
$K \subset N_0$	With $k, k_0 \in K$ and $k \leq k_0$
$N \subset N_0$	With $n, n_0 \in N$ and $n \leq n_0$
(N, d)	Metric space on the set *N* with metric *d*
σ	Electrical conductivity
$\Omega_u \subset R^4$	Technological solution space with $\boldsymbol{u} \in \Omega_u$
$\Omega_{u_0}, \Omega_{u_I}, \Omega_{u_{II}}$	Subsets of the technological solution space
LE	Unit of length
r_s	Substantial risk factor
r_1, r_2	Sub-risk factors
μ	Failure factor
p_i	Failure likelihood of the *i*th microcell
P_{i_0}	Failure likelihood of a macrocell with i_0 users
$\boldsymbol{S}^{**} \in R^6$	Extension of the structure vector S^* with the substantial risk factor
S_6	Structure variable for the substantial risk factor
v_{tv}	Availability
$v_{tv,\,B}$	Sustainability boundary; sustainable availability boundary
$v_{tv} \cdot E^D_{\text{Nat.}}(t)$	National sustainability
$v_{tv,n} \cdot E^D_n(t)$	Regional sustainability in a macrocell
$\boldsymbol{v}$	State vector with sustainability component
$\Omega \in R^5$	Sustainable technological solution set with
$E^N_{T_{\text{Ref.}}}$	Annual energy demand in a reference year
λ^-_i	*i*th supply factor
$\lambda^-_{\min}$	National supply factor
$h(t)$	Distribution of annual energy demand with $\int_{t_0}^{t_0+365} h(t)dt = 1$
$g(x) \in [0; 1]$	Weights between λ^-_{min} and r_s
$E^D_{T_{\text{Ref.}}}$	Annual reference energy arbitrary initial value $\int_{t_0}^{t_0+365} E^D(t)\,dt$

Appendix 2

Table 4 Follow-up themes

Social opportunities and risks of pervasive use of electrical power
Gross domestic product: country-specific direct and indirect effects of electrical energy for products and services
Entropy of national electrical energy supply systems
Social effects from municipalization/nationalization responses to technological and economic costs in the national EESS
Quantitative determination of a sustainability boundary for the national EESS
Quantitative determination of a weighting function for operational supply energy values and technological system availability
A basis for user decisions on the establishment of national contributions
Sociological effects of electricity storage in national EESS
Technological effects of electricity storage in national EESS
Incorporating user requirements into national legislation
Timing and process of transformation of a national EESS

Supplementary information

Supplementary information accompanies this paper at https://doi.org/10.1186/s13705-019-0221-4.

Additional file 1. Mathematical and physical foundation of grid structure.

Additional file 2. State vectors and grid spectra.

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