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On the axiomatic foundation of carbon and energy footprints

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Abstract

Background: A wide variety of footprint concepts is proposed in the scientific literature giving rise to a selection problem.

Method: The objective of this paper is to provide an axiomatic foundation to the concept of ecological footprint indices, in particular carbon and energy footprints. For this purpose, we define five axioms representing general properties which any ecological footprint measure should fulfill.

Results: It can be shown that there exists a unique index which is characterized by the given set of axioms. Its functional form is determined, and an economic interpretation is given. The most prominent empirical application is discussed.

Conclusion: We find that the proposed index as a generalization of more specific indices like carbon and energy footprint indices may confirm some important issues discussed in the literature. First, it incorporates a trade component indicating the ecological footprint of economic activities embodied in the trade pattern of a country or region. Moreover, the productivity of land use in production as a means to mitigate the pressure on the ecological system is reflected. But, most importantly, from a methodological point of view, there is no longer the need for designing ecological footprint indices ad hoc, in particular for the sake of empirical application.

Keywords: Ecological footprint indices, Axiomatic foundation, Sustainability indicators, Sustainable welfare

PACS: JEL classification: Q01, Q20, Q30, C43

Background

Introduction

Over the last three decades, sustainability has become a widely accepted and most important objective in policy. For measurement of sustainable utilization of the biosphere and its resources, different approaches have been developed to obtain qualitative and quantitative conclusions. One of these concepts is called the ecological footprint. It indicates the "pressure" on the natural resources exerted by the population of a region or country and its economic behavior, in particular by means of land-area use.

For example, [1], regarded as one of the pioneer works, and, therefore, one of the foundations of the ecological footprint concept, find that this index can offer an estimation of the land use for consumption activities of humans as well as for the assimilation of waste products by nature. The general application proposes that the more land area is required by human activity or consumption, all other things being equal, the higher is the pressure on the biosphere.

Hence, one of the major assumptions underlying the ecological footprint concept is that future global welfare highly depends on meeting the strong benchmarks of sustainability. As such, it calls for maintaining the biosphere as a source for any kind of economic activity as well as a sink for the waste products generated alongside. Given that the consumption is largely driven by the availability of

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renewable resources [2], the debate on sustainable lifestyle is strongly connected to the debate on climate change [3]. The ecological footprint concept thus generates a link between sustainability and consumption activities. In particular, this is true for carbon and energy footprints.

Further research on ecological footprints and its controversial debate in the literature created a broad variety of indices proposed for measuring a specific footprint (e.g., [4–7] and [8] provide a comprehensive overview and well-founded comparison of different sustainability indicators). However, to our knowledge, an axiomatic foundation to specify the characteristics considered as being appropriate for ecological footprint measures in general has not been proposed yet. Therefore, to this day, ecological footprint indices primarily have been developed through the adaption of the groundbreaking concepts provided by Wackernagel and Rees (i.a. [1–4, 6, 7, 9–15] and [16]). Their axiomatic characterization, however, may still be investigated.

Literature

The origin of ecological footprint indices is an academic work by Rees [9]. Initially, the term "carrying capacity" was first defined as the rate of sustainable consumption and waste discharge in a given region. The term "ecological footprint" refers then to the land area needed to sustain a certain material standard of living. At that time, disputes arose how area is used as a variable to cover the demands of an urban region. Before, urbanization focused solely on the area used for living and providing goods and services for the citizen inside the urbanized structure [3]. Rees [9] argues that urban economies actually need far more area to sustain their material standard of living, and thus, to satisfy their demands, goods have to be traded from and to other regions. Cities appropriate effectively carrying capacity from afar. Therefore, the area needed for production elsewhere has to be included when researching urbanization. This statement was a clear distinction compared to common research before.

Later on, the purpose of the ecological footprint changed such that it measured the burden imposed on the biosphere by human activity [10]. They argue that the impact of an activity on nature correlates with the area needed to perform it. That means the used natural capital needs to be expressed in (bio-)physical units in order to enable a quantitative measurement. The authors emphasize that a monetary analysis is inappropriate because it implies substitutability, enables future-related discounting, and examines marginal rather than absolute values. Moreover, they state that their footprint measure is determined by basically four variables, namely the ecological productivity, the population size, living standards, and technology. The ecological footprints of people cannot overlap which indicates a competition for ecological space [10]. According to these considerations, the ecological footprint of a country or region, EF, is calculated as the sum of the footprint components of all N sectors in a country or region multiplied by a so-called equivalence factor EQF_i:

$$EF = \sum_{i=1}^{N} FC_i \cdot EQF_i \tag{1}$$

where FC_i is the footprint component of sector i ($i = 1, \ldots, N$) determined by

$$FC_i = \frac{Y_i}{\gamma_i^n} \frac{\gamma_i^n}{\gamma_i^w} \tag{2}$$

$$\gamma_i^w = Y_i^w / A_i^w \tag{3}$$

$$\gamma_i^n = Y_i^n / A_i^n \tag{4}$$

with Y_i denoting the product output of sector i, A_i the land area appropriated by sector i, γ being the average product yield [10], and n, w stand for a country and the world, respectively. The ecological footprint is typically measured in global area units (e.g., [5, 8] and [4]). A country's footprint can thus be interpreted as the total land-area appropriated to output production at world average productivity [10]. If the numerator is being replaced by $C_i + XS_i$ with XS_i denoting a country's net exports in sector i, we can easily derive the ecological footprint of consumption. This index offers insights on how much area is used for the specific consumption of one region, and it is therefore regarded as an indicator of consumption-based accounting.

While the ecological footprint focuses mainly on the pressure of human demand on the biosphere's regenerative capacity, other footprint measures have been developed to specifically examine the impact of human activity on certain ecosystem compartments, the carbon footprint, and the closely related energy footprint being one of the more important examples.

In the context of the footprint literature, the carbon footprint indicates how much area is needed to sustainably absorb a certain rate of carbon-equivalent emissions of certain activities [17]. The energy footprint on the other hand measures the land area needed to sustain a certain type of energy consumption or production and includes the carbon footprint as well the land area needed to sequester other kinds of emissions like nitrogen and sulphur [18].

Therefore, the notion of the footprint family has been recently introduced in the literature. Although there are indeed overlaps between the footprint approaches which give rise to concerns of double counting, most authors recognize a certain complementarity within the footprint family (e.g., [5, 8] and [4]) and suggest a joint application of the ecological, carbon, and water footprint with the ecological footprint being narrowed down to a land footprint to avoid double counting.

A different approach constitutes the Sustainable Process Index (SPI) which was developed by [19]. It evaluates the operability of economic activity based on a life-cycle assessment, provided that sustainability is ensured. The SPI is a part of ecological engineering, and it is argued that sustainability of the economy is achieved if a sustainable flow of solar energy is provided. However, area is required for the conversion of solar energy into services and is thus a limiting factor. The SPI is calculated as the ratio of the specific service area, A_{tot}/S_{tot}, and the per-inhabitant area in the region which is relevant for the production process, a_{in}:

$$SPI = \frac{A_{tot}/S_{tot}}{a_{in}} \tag{5}$$

with A_{tot} denoting the total area assigned to embed a process sustainably in the ecosphere and S_{tot} denotes the number of unit-services such as the units of output which are supplied by the process within one reference period. Measuring the area needed to provide the necessary material and energy proportional to the available area for the economic activity of one human being is the procedure of the SPI [19]. Given that the area is included in the numerator as well as in the denominator, the SPI is expressed in capita per service unit. Once again, the main variables are area and energy related to population size similar to Wackernagel's approach by [10].

Another approach is the component-based model of ecological footprinting developed by [20] as an alternative ecological footprint. The purpose of study was to calculate the impact of different human lifestyles and organizations on nature on the domestic level. Hence, it is a bottom-up approach usable for small entities such as households, organizations, and regions within nations. The calculation of the component-based ecological footprint, CBEF, itself is similar to the footprint of [10], but the area use and economic process get subdivided into smaller (sector-specific) parts as long as data is available ([20]):

$$CBEF_i = \frac{\sum_X A_X}{Y_i} \tag{6}$$

where A_X denotes the area needed for the production of the intermediate good X which is used for the production of the final product Y, and Y_i indicates the service quantity of the final product Y in (service) sector i. Consequently, the component-based subindex is typically expressed in area per functional service unit such as hectares per passenger kilometer. The main variables are land use for the production of the intermediate, a final good, and a region for which the analysis is performed. These subindices can then be aggregated over different consuming entities (e.g., a population, organization, or region) with given amounts of consumption c_i of service or product i

$$CBEF = \sum_i CBEF_i \cdot c_i \tag{7}$$

Other indicators have been proposed in the ecological literature to evaluate sustainability and environmental services. They include the emergy analysis developed by Odum [21, 22], the IPAT equation which resulted from the debate between [23] in the early 1970s and had later been refined by [24], and the set of socioeconomic metabolism core indicators drawn up by Fischer [25, 25, 26]). Expressing the amount of energy used in work processes to make a product or service, *emergy*—which is the short term for "embodied energy'—accounts for quality differences between different forms of energy (e.g., sunlight, water, minerals, or fossil fuels) which are generated by transformation processes in nature. Emergy is typically reflected in "unit emergy values" which represents the emergy required to generate one unit of output[1] or as a flow, i.e., emergy released or used per time unit, called "empower" ([21, 22, 27]). From there, a comprehensive nomenclature of emergy-based indicators has emerged, focusing on information related to various dimensions such as space, time, or money, as well as the performance with regard to them (see [28, 29], among many others).

In contrast, the *IPAT* equation (or, put mathematically, $I = P \times A \times T$) describes a formula in which the human impact on the environment (I) is equated to the product of the factors population (P), affluence (A), and technology (T) [23, 30]. However, its use as an indicator for the anthropogenic impact on nature has been quite limited, given the extensive criticism on its simplicity[2] and the impossibility of hypothesis testing [3]. After being reformulated as the *ImPACT* equation[4], this accounting equation has been transferred into a stochastic model, called the *STIRPAT* model for analyzing stochastic impacts by regression on each of the key variables [24, 34–36].

[1] In this regard, emergy is often measured either per unit money or per unit labor [22].
[2] Many authors deprecate the neglect of interdependencies among the factors considered as well as of other factors such as political and social structures or beneficial human impacts (e.g., [31–33] and others). Others find themselves at odds with the proportionality in the functional relationship between the factors ([34]).
[3] This is due to the fact that known values of some terms determine the value of the missing term ([34]).
[4] This was actually done to better identify factors that can be altered (e.g., by policy) to reduce impacts by disaggregating T into consumption per unit of GDP, C, and impact per unit of consumption, T which led to the new equation $I = P \times A \times C \times T$ ([34])

Finally, within the framework of *material flow accounting (MFA)*, a large number of indicators have been established to measure environmental pressures caused by human activity which are commonly formulated as material intensity (or, conversely, material productivity) or "metabolic rate"[5] which indicates the annual material flow per capita population. They all aim at measuring society's environmental performance and efficiency in terms of the overall material and energetic turnover of national economies [25, 25, 26].

Our approach

The reasoning for the mathematical appearance of the various footprint indices is often alike with certain existing similarities related to the variables employed. It is undisputable that the intended ecological interpretation is one important reason for the defined structure of such indices. Yet, none of the footprint measures used in the literature have been further reviewed for their axiomatic foundations while the majority of economic indices such as price indices or indices of economic inequality is thoroughly analyzed and is subject to general mathematical principles. This fact is all the more surprising considering that ecological footprint data is commonly reflected in the official statistics of various international organizations and have been included in those institutions' methodologies on environment statistics as an indicator for sustainable land use (see, e.g., [37–40]). As a consequence, the appropriateness of ecological footprint indices is still to be discussed (a controversial debate has been initiated by [41] and [13], among many others) and established as far as their axiomatic characterization is concerned.

This paper therefore intends to provide an axiomatic approach to a general ecological footprint measure. By analyzing the features which footprint indices proposed in the literature might have in common, their appropriate characteristics (such as variables and their interdependencies) are identified. Then, we define a set of fundamental axioms representing the theoretical concept we have in mind of what an ecological footprint should measure. The advantage of this approach lies in the proposition of a few stylized facts for which we have a clear idea on how the index should respond to. But most importantly, once the formula of the index has been determined, it can be applied to any real-world situation, irrespective of the particular values the independent variables may take.

Following this procedure, we can show that a unique index exists which meets the axiom system proposed. We find that the index characterized is resolving some important issues discussed in the literature. First, it incorporates the impact of a country's trade pattern on the sustainability of consumption and human economic activities. In a sense, the footprint embodied in the imports and exports of goods and services is accounted for. Moreover, the importance of land productivity as a means to mitigate the pressure on the ecological system is reflected by the measure. But most importantly, from a methodological point of view, ecological footprint measures are given a theoretical foundation in a fairly general case.

This paper is of particular interest to those researchers who are directly involved in indicator development and may contribute to empirical research in the future. Hopefully, our results can be used whenever modifications of currently used indices may be called for.

The outline of the paper is as follows: in the "Methodology" section, we try to give a short review of the axiomatic method. From there, in the "Results" section , we are able to emphasize some particular properties of economic footprint measures, represented by an axiom system which any index should meet. Finally, the derived index is discussed and compared with existing footprints in the "Discussion" section. The "Conclusions" section concludes the discussion.

Methodology

Given the different approaches to measure a relative concept which have been customized to their respective research focus as well as to their field of application, it is essential to create an axiomatic foundation to reveal the properties an appropriate consumption-based footprint index should exhibit, in particular with respect to the functional form mapping the input variables. Thereby, it could not only be assessed whether the footprint indices proposed in the literature satisfy the properties considered but, above all, how an appropriate index should look like mathematically. Our analysis thus should address such an axiomatic system in the following section.

The axiomatic-deductive methodology has a long history as it is one of the oldest scientific methods going back as far as ancient times. It was most prominently employed by Euclid to put the mathematics of his time on a sound foundation with a long lasting influence even up to this day, and by Newton with his Principia Mathematica in physics. Another prominent example is Von Neumann and Morgenstern with their groundbreaking work Theory of Games and Behavior [42]. From then on, the axiomatic method was firmly established in economics and was used to ground such diverse fields as social choice [43–45], price indices [46], or inequality indices like the Gini-Index [47, 48], but also environmental indices [49].

The major advantages of such an approach are that the discussion is made precise and explicit to a degree that is not possible to attain using only verbal means, where implicit assumptions and value judgements can creep into

[5] It expresses the average amount of material associated with sustaining one individual during a year ([25]).

the discussion unnoticed. As such the axiomatic method is certainly a part of the toolkit in the "the pursuit of objectivity" as mentioned in [50].

Another advantage is that the focus is put on the definition of the footprint concept itself according to the idea of what a footprint should measure, irrespective of the issue of its operationalization. While the procedure to operationalize the variables that go into the footprint measure is carried out more specifically, the functional form of the index itself is gaining general validity.

Results

The axiom set

As argued above, there is a need for an axiomatic foundation of the ecological footprint concept. Hence, it is to be determined which properties such an index must meet for proper empirical applications to measuring the sustainable land use within an economy, like, specifically, sustainable energy use. In this section, we therefore develop a system of axioms for ecological footprint indices (EF) comprising properties generally accepted in the relevant literature. First, we have to give a general definition.

Definition 1 *Let $D = \mathbb{R}^5_+$ be a set of ecological states, where $x = (C_n, Y_n, A_n, Y_r, A_r) \in D$ is a vector comprising national product consumption C_n, national product output Y_n, national land area A_n appropriated, product output in the rest of the world Y_r, and land area used in the rest of the world A_r.*

For the energy footprint, the product output and the product consumption are defined as energy, and the land area appropriated is defined as the area needed for energy production and sequestration of the energy-related emissions. In the case of the carbon footprint, the product is defined as carbon sequestration (supplied by the ecosystem), product consumption as carbon emissions (being a demand on the ecosystem), and the land area as the area needed to provide the sequestration capacity.

Then, the ecological footprint index is a mapping:

$$f : D \mapsto \mathbb{R}_+, \quad x \mapsto f(x)$$

with the meaning that the ecological footprint of ecological state x is bigger than that of y if $f(x) > f(y)$

The function f should satisfy the following set of axioms.

Axiom 1 (Monotonicity) *The function f is strictly increasing in C_n:*

$$f(\underline{C_n}, Y_n, A_n, Y_r, A_r) < f(\overline{C_n}, Y_n, A_n, Y_r, A_r) \quad for \quad \underline{C_n} < \overline{C_n}$$

Remark 1 *It seems to be natural to assume that the ecological footprint index should take higher values if a country's product consumption is going up, all other things equal. This property is evident in what an EF should measure from the view of sustainable land area use: It indicates that there is a higher demand on renewable natural resources used in production, thus increasing the burden on the eco-system.*

Axiom 2 (Commensurability in Consumption) *If there is an equally proportional change in a country's consumption and the world production output, all other things equal, then the value of the ecological footprint remains the same:*

$$f(\lambda C_n, \lambda Y_n, A_n, \lambda Y_r, A_r) = f(C_n, Y_n, A_n, Y_r, A_r) \qquad for \quad \lambda > 0$$

Remark 2 *Since the proportionally increasing world output is being produced using the same world land area as before, a country can increase its consumption in the same proportion without increasing its demand on the worldwide bio-resources. In this case, the value of the index should not change. This axiom may also reflect the impact of the land productivity in agricultural production on the ecological system; however, only if the land-use intensity is sustainable. Thus, "all other things equal" would include environmental pressures (i.e., land area needed for sequestration of emissions). The implicit assumption here is that the emission efficiency of energy production is higher due to technical progress. The case where environmental pressure is rising is reflected by the following axiom.*

Axiom 3 (Proportionality to Land Area Use) *The index is directly proportional to the world land area appropriated, all other things equal.*

$$f(C_n, Y_n, \lambda A_n, Y_r, \lambda A_r) = \lambda f(C_n, Y_n, A_n, Y_r, A_r) \qquad for \quad \lambda > 0$$

Remark 3 *Since the world's bio-resources are spatially distributed, an increase in the worldwide land area used implies that the demand on the carrying capacity of the ecological system to satisfy the same consumption needs is increasing. In this sense, this axiom is perfectly in line with the spirit of what the ecological footprint index should reflect.*

Axiom 4 (Commensurability in Production) *An equally proportional change in the world land area and the world production does not change the value of the index*

$$f(C_n, \lambda Y_n, \lambda A_n, \lambda Y_r, \lambda A_r) = f(C_n, Y_n, A_n, Y_r, A_r) \qquad for \quad \lambda > 0$$

Remark 4 *This axiom is proposing that the ecological footprint should not respond to a directly proportional change in the world land area used and the world production, all other things equal. In this case, the world average product yield remains the same for any given amount of national consumption. It means that a country's product consumption requires the same share of the earth's ecological resources, such that the value of its footprint should remain constant. In other words, only if the extended land area is entirely devoted to production, and not to the sequestration of emissions, the index should remain unchanged. In any other situation, the index would have to change, for instance according to axiom 3.*

Axiom 5 (Compensability) *A shift in the land area use between countries as well as a shift in production output between countries does not change the value of the index. More precisely, the footprint remains unchanged if a change ΔY in national output produced is offset by an equal and opposite change $-\Delta Y$ in the output produced in the rest of the world. And, at the same time, a change in the national land area use ΔA is offset by an opposite change in the land use $-\Delta A$ in the rest of the world:*

$$f(C_n, Y_n + \Delta Y, A_n + \Delta A, Y_r - \Delta Y, A_r - \Delta A) = f(C_n, Y_n, A_n, Y_r, A_r)$$
$$\text{for} \quad Y_n \geq -\Delta Y \wedge Y_r \geq \Delta Y \wedge A_n \geq -\Delta A \wedge A_r \geq \Delta A$$

Remark 5 *This axiom is related to the previous axiom, but with the distinction that in this case the world average product yield stays the same through offsetting changes in the national production and national land use by complementary changes in the rest of the world. So, from a global view, the demand of national product consumption on the worldwide natural resources remains the same. The only difference is the absence of any scale effect in world production and world land area. However, since such scale effects should not affect the index anyway according to axiom 4, the same line of reasoning as before applies here.*

In the following section, we will propose an index which satisfies the five axioms simultaneously.

Existence and uniqueness of the ecological footprint index

We will now state the following proposition.

Theorem 1 (Existence and Uniqueness) *Axioms 1 through 5 characterize the following unique index up to a strictly positive arbitrary coefficient which is $k > 0$:*

$$f(C_n, Y_n, A_n, Y_r, A_r) := k \frac{C_n}{Y_n + Y_r} \cdot (A_n + A_r)$$

Proof See Appendix. □

Please note, the index is unique up to a constant coefficient k which can be chosen arbitrarily from $\mathbb{R}_+$, the positive set of real numbers. This factor gives the unit of the scale, i.e., the norm.

As an implication, the scale of the index is determined as a ratio scale since any transformation of the scale by an arbitrary factor is feasible. In this case, the ratio of scale values remains invariant. Therefore, the information content of the scale is given by the ratio between any two values.

Discussion

The index proposed above may have different meanings, irrespective of setting the norm k, and its functional form can be read in different ways, which are equivalent. First, it may be interpreted as the land area appropriated to provide a country's share of consumption on world production, C_n/Y_w with $Y_w := Y_n + Y_r$ and $A_w := A_n + A_r$.

$$f = k \frac{C_n}{Y_w} \cdot A_w \tag{8}$$

Secondly, the index indicates how much land area of world average productivity is appropriated to satisfy a country's consumption needs.

$$f = k \frac{C_n}{Y_w / A_w} \tag{9}$$

Finally, the index is measuring a country's consumption share of the production in terms of the national land area used, weighted by the relative national product yield.

$$f = kA_n \cdot \frac{C_n}{Y_n} \cdot \frac{Y_n / A_n}{Y_w / A_w} \tag{10}$$

As far as the issue of imports and exports is concerned, we consider the case of $k = 1$ and $C_n = Y_n - XS_n$ with national net exports XS_n. Then, we get with $\gamma_w := Y_w/A_w$:

$$f = \frac{(Y_n - XS_n)}{\gamma_w} \tag{11}$$

Therefore, the footprint of an exporting country, $XS > 0$, is smaller then the footprint of an importing country, $XS < 0$, all else equal. This could be seen as a possibility for decoupling a country's footprint from its production. However, in fact, the ecological footprint is conditioned on consumption rather than on production. The idea behind is to attribute the resource use to the countries where consumption finally takes place.

Let us provide an example as to how the index derived above can be used in an applied context. Since the matter of operationalization of the constituent variables in the domain of the index would involve a discussion of complex physical, chemical, and technical relationships beyond the scope of this paper, we will use an example from the literature [51] where some simplifying assumptions are made. There, a carbon footprint EF_C is calculated in terms of forest area needed to sequester worldwide carbon emissions. To be more precise, the calculation runs as follows[6].

$$EF_C = \frac{P_C(1 - S_{OCEAN})A_F}{NFP/0.27}$$

where

- P_C are the global CO_2-emissions (in Mt CO_2 per year).
- S_{OCEAN} is the fraction of CO_2 sequestered by the oceans, which is 28% for the year 2010.
- A_F is the total amount of forrested area (in ha).
- NFP (Net Forest Production) is the total amount of carbon captured in biomass by forests (in t C per year).
- 0.27 is the share of carbon in CO_2 by weight.

Taking an environmental perspective, we can obtain this index from the index f above by setting the product consumption $C_n = P_C(1 - S_{OCEAN})$, (i.e., CO_2-emissions placing an additional demand on ecosystem services). The national production Y_n will be set to $NFP/0.27$ (carbon dioxide sequestered in biomass), and the land area needed A_n for the sequestration of carbon to A_F. Finally, Y_r, A_r will have to be set to 0, because the region under consideration is already the whole world.

The value NFP/A_F was calculated in [51] to be 0.73 tonne carbon per hectare and year. [51] also cites a value of 38.7 Gigatonnes of worldwide anthropogenic carbon emissions for the year 2010. By substituting these values into the formula above, we get a world carbon footprint as measured in forest area needed of 14.31 Gigahectare.

With respect to the indices mentioned in the introduction, we find by inspection of the respective algebraic expressions that the axioms stated above characterize a component FC_i of the Wackernagel index, i.e., the carbon component. This result can most directly be obtained by comparing Eq. (8) with Eqs. (2) and (3). After substituting (3) into (2) and simplifying the fraction (also, formally removing the subindices i), we get essentially the same expression except for the arbitrary scaling factor k.

Regarding the SPI, the component-based ecological footprint CBEF, and the IPAT/ImPACT-equation, we shortly note that they do not meet the axiom set. For instance, these indices, as being purely production based, do not include a consumption variable as required by the axioms. Further, the CBEF is defined as an aggregation of intermediate products which in turn are not captured by the axiomatically characterized index. And finally, the population variable is a factor in the IPAT-index, which cannot be motivated by the axiom system. However, to be fair, this result does not invalidate these indices, since they may possibly be also derived from some set of axioms, still to be investigated. Likewise, the axiom system given may not be the only one possible from which the index stated above can be derived.

Conclusions

The purpose of this paper has been to establish an axiomatic foundation to the concept of ecological footprint indices. We first identified the characteristics of a wide variety of indices proposed in the literature and discussed the relations between the various input variables. We then set up five axioms which we considered appropriate for footprint indices in general. It has been shown that there exists a unique index which is meeting the set of axioms simultaneously, and its functional form has been derived. In particular, we have characterized carbon and energy footprint indices as components of the compound based footprint measure.

We find that the proposed index is resolving some important issues discussed in the literature. First, it incorporates the country's trade pattern. Moreover, the importance of land productivity as a means to mitigate the pressure on the ecological system is reflected. From a methodological point of view, an axiomatic foundation is provided to ecological footprint measures in a fairly general case. This is of particular interest for empirical applications.

However, in our belief, a future field of research will open up for examining the potential existence of even more footprint indices and their properties to be based on alternative axiom systems. In this line, the index proposed in this paper may serve as just one example of measures following reasonable mathematical postulates. Moreover, the aggregation procedure for the components is still open to discussion, since the restrictive construction of a common scale like global hectares might be dispensable.

Appendix

Proof First, it can be easily seen that $f(C_n, Y_n, A_n, Y_r, A_r) := k\frac{C_n}{Y_n+Y_r} \cdot (A_n + A_r)$, with $k > 0$ satisfies axioms 1 to 5. This is shown by a straightforward calculation after substituting EF for f in the axioms, which proves the existence of an index that satisfies the given axioms.

[6] We combine here Eqs. (3) and (4) of [51]. EQF can be dropped, as it can be seen as the scaling factor k. Note that Eq. (3) contains a typographical error in the original. There ,P_C is multiplied by S_{OCEAN} rather than $(1 - S_{OCEAN})$.

It remains to show the uniqueness of the index. For this, we will derive *EF* from axioms 2 to 5.

By axiom 5, we have with $\Delta Y = Y_r$ and $\Delta A = A_r$

$$f(C_n, Y_n, A_n, Y_r, A_r) = f(C_n, Y_n + \Delta Y, A_n + \Delta A, Y_r - \Delta Y, A_r - \Delta A) = f(C_n, Y_n + Y_r, A_n + A_r, 0, 0)$$

Continuing by setting $Y_w := Y_n + Y_r$ and $A_w := A_n + A_r$ and applying axiom 2 with $\lambda = 1/C_n$, we obtain:

$$f(C_n, Y_w, A_w, 0, 0) = f(1, \frac{Y_w}{C_n}, A_w, 0, 0)$$

Similarly axiom 3 with $\lambda = \frac{C_n}{Y_w}$ applied to the RHS of the last equation yields

$$f(1, \frac{Y_w}{C_n}, A_w, 0, 0) = f(1, 1, \frac{C_n}{Y_w} A_w, 0, 0)$$

Finally, using axiom 4 with $\lambda = \frac{C_n}{Y_w} A_w$ and one finally gets

$$f(1, 1, (\frac{C_n}{Y_w} A_w) \cdot 1, 0, 0) = \frac{C_n}{Y_w} A_w f(1, 1, 1, 0, 0)$$

Comparing this to the first equation, we have

$$f(C_n, Y_n, A_n, Y_r, A_r) = \frac{C_n}{Y_w} A_w f(1, 1, 1, 0, 0)$$

Clearly, if this is to be a monotonically increasing function in C_n as stated by axiom 1, $k := f(1, 1, 1, 0, 0)$ has to be positive (Y_w, A_w being positive).

Thus, axioms 1 through 5 define the unique index $f(C_n, Y_n, A_n, Y_r, A_r)$ up to a multiplicative factor $k > 0$. □

Abbreviations
CBEF: Component-based ecological footprint; EF: Ecological footprint index; MFA: Material flow accounting; SPI: Sustainable Process Index

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Authors' contributions
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